



## Measurement of $R = B(t \rightarrow Wb) / B(t \rightarrow Wq)$ in the *Sec Vtx* tagged lepton plus jets sample with $8.7 \text{ fb}^{-1}$ of Data

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In this analysis we measure the ratio of the branching fractions  $R = B(t \rightarrow Wb) / B(t \rightarrow Wq)$  in the lepton plus jets channel using the full Run II dataset corresponding to  $8.7 \text{ fb}^{-1}$  of data collected by the CDF II detector.  $R$  is obtained by maximizing a likelihood while leaving the  $t\bar{t}$  cross section as a free parameter in a recursive procedure. We measure  $R = 0.94 \pm 0.09$  (stat+syst) and  $\sigma_{p\bar{p} \rightarrow t\bar{t}} = 7.5 \pm 1.0$  (stat+syst) pb. Assuming the unitarity of the CKM matrix and three generation of quarks we extract  $|V_{tb}| = 0.97 \pm 0.05$  (stat+syst), in agreement with the Standard Model expectations. Under the same assumptions we set a lower limit  $|V_{tb}| > 0.89$  at 95 % C.L.

## I. INTRODUCTION

In the Standard Model the top quark decay rate is proportional to  $|V_{tq}|^2$ , the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. Since the assumption of three generation of quarks and the unitarity of the CKM matrix lead to  $|V_{tb}| = 0.99915_{-0.00005}^{+0.00003}$  [1], it can be assumed that top quark decays exclusively to  $Wb$ . On the other hand, if more than three generation of quarks are allowed, the constraint on  $|V_{tb}|$  is removed and lower values are possible, affecting top cross section measurements, B mixing and CP violation.

A direct measurement of  $|V_{tb}|$  matrix element can be obtained by measuring the single top production cross section, but a value can be extracted also from the top quark decay rate in the  $t\bar{t}$  channel. It is possible to define  $R$  as the ratio of the branching fractions:

$$R = \frac{\mathcal{B}(t \rightarrow Wb)}{\mathcal{B}(t \rightarrow Wq)} = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2} \quad (1)$$

expected to be  $0.99830_{-0.00009}^{+0.00006}$  under the previous constraints.

## II. OVERVIEW

In this note we present an update of the measurement of  $R$  [2] using a data sample corresponding to  $8.7 \text{ fb}^{-1}$  collected by the CDFII detector at  $\sqrt{s} = 1.96 \text{ TeV}$ . The analysis is performed in the lepton plus jets channel, where one  $W$  boson, coming from  $t\bar{t} \rightarrow W^+qW^-\bar{q}$ , decays hadronically while the second decays in a charged lepton and a neutrino.

CDF performed several measurements of  $R$  both during Run I and Run II, combining the l+jets channel with the dilepton channel. The most recent result measured  $R = 1.12_{-0.19}^{+0.21}(\text{stat})_{0.13}^{+0.17}(\text{syst})$  using an integrated luminosity of  $162 \text{ pb}^{-1}$ ,  $R > 0.61$  at 95% CL [2]. The DØ collaboration has measured recently  $R$ , using  $5.4 \text{ fb}^{-1}$ , with a simultaneous fit on the top pair production cross section, in the l+jets and dilepton channels. Their result is  $R = 0.90 \pm 0.04(\text{stat}+\text{syst})$  and  $R > 0.79$  at 95% CL [3]. Since the uncertainty on the previous CDF measurement was dominated by the statistics, we performed a new measurement with the full CDF dataset.

The measurement is based on the determination of the number of b-jets in  $t\bar{t}$  events using the lepton plus jets sample with at least three jets in the final state. We consider events in which the charged leptons are either electrons or muons. Identification of jets coming from b-quark fragmentation (b-jet *tagging*) is performed by the *SecVtx* algorithm, based on the reconstruction of displaced secondary vertices. We divided our selected sample in subsets according to the type of lepton, number of jets in the final state and events with one or two tags. The comparison between the total prediction, given by the sum of the expected  $t\bar{t}$  events and background estimate, and the observed data in each subsample is made using a Likelihood function. In the current analysis we are fitting simultaneously  $R$  and the top pair production cross section  $\sigma_{t\bar{t}}$ , through a recursive fit.

## III. Data Samples and Event Selection

The signature of our signal is constituted by a high  $p_T$  charged lepton, large missing transverse energy  $\cancel{E}_T$ , due to the presence of an escaping neutrino from the leptonic W decay, and jets.

Data are collected by the three-level trigger system by the following paths: an inclusive lepton trigger that requires an electron (CEM) or a muon (CMUP or CMX) with  $E_t(P_T) > 18 \text{ GeV}$  (GeV/c).

We further purify this set by applying offline the following requirements:

- **Lepton:** we require one and only one *tight* lepton: a CEM electron with  $E_T > 20 \text{ GeV}$  and  $|\eta| < 1.1$  or a CMUP or CMX muon with  $p_T > 20 \text{ GeV}/c$ .

As leptons in  $W$  decays are expected to be isolated, we also apply the following requirement:  $E_T$  not assigned to the lepton in a cone of  $R < 0.4$  around the lepton direction must be  $< 10\%$  of the lepton  $E_T$ .

- **Jets:** are reconstructed offline by a fixed cone algorithm with a radius  $R=0.4$ . Jets are required to have  $E_T > 20$  GeV and  $|\eta| < 2.0$ . The jet energy is corrected for detector non-uniformities, cracks, overlapping events and parton-fragmentation effects [7].

In order to further reduce the multijet background we require  $E_T > 30$  GeV for the most energetic jet and  $E_T > 25$  GeV for the second one.

We accept events with at least three tight jets in the final state.

- **Missing Transverse Energy:** require  $\cancel{E}_T > 20$  GeV (after correcting for the presence of muons and for the Jet Energy Scale) to account for the presence of a  $\nu$  from the  $W$  decay.
- **b-tagging:** is needed to determine the number of b-jets in each jet-bin. The *SecVtx* algorithm is used to tag heavy flavor jets. The tagging efficiency in data and in MC is different, therefore we apply a Scale Factor (SF)  $0.96 \pm 0.05$  to MC events.

- **$W$  Transverse Mass:**

$$M_T^W = \frac{1}{c^2} \sqrt{2 \cdot E_T^l \cdot \cancel{E}_T \cdot (1 - \cos \phi_{l\nu})} \quad (2)$$

where  $E_T^l$  is the transverse energy of the lepton and  $\phi_{l\nu}$  is the angle between the lepton and the  $\cancel{E}_T$ . We reject events with  $M_T^W < 20$  GeV  $/c^2$ .

- **Vetoos:** additional vetoos are added to our selection to exclude dilepton events,  $Z$  candidates and cosmic rays.

## V. Sample Composition and Background Calculation

The composition of our data sample, including the estimate of the backgrounds, is obtained using a standard CDF algorithm [8] It was developed to estimate the composition of the *SecVtx* tagged lepton plus jets data sample. This tool is used in several CDF analyses, i.e. the  $t\bar{t}$ , the *single top* [9] or  $WW/WZ$  [10] cross section measurements.

In our analysis the sample is composed by the  $t\bar{t}$  signal and the following backgrounds:

- **Electroweak processes:** this background is due to contributions coming from known electroweak processes characterized by well predicted production cross section and branching ratios.
- **W+jets:** this background is composed by the production of a real  $W$  in association with high energy jets. This background can be divided into W+HF (heavy flavor) and W+LF (light flavor), that can affect the tagged sample because of jets mistakenly identified as heavy flavor jets by the *SecVtx* algorithm.
- **Non-W:** this background is due to QCD processes that can fake a W+jets signature. A high energy jet can fake a high- $P_T$  lepton, especially an electron, and the  $\cancel{E}_T$  can be faked by an erroneous reconstruction of the total transverse energy of the jets.
- **Z+jets:** this background is composed by the production of a real  $Z$  boson in association with high energy jets and it is described by an accurately measured cross section.

The normalization of known processes is calculated using the production cross section, the integrated luminosity of the sample, the trigger efficiency, the overall selection acceptance and the tagging efficiency. The yields are given by:

$$N_{p\bar{p} \rightarrow X} = \sigma_{p\bar{p} \rightarrow X} \cdot \epsilon_{evt} \cdot \epsilon_{tag} \cdot \mathcal{L} \quad (3)$$

where:

- $X$  is the considered process ( $t\bar{t}$ , single-top,  $WW$ , etc.)
- $\sigma_{p\bar{p} \rightarrow X}$  is the production cross section [11].

- $\mathcal{L}$  is the integrated luminosity. Our calculation takes into account that the collected luminosities for CEM, CMUP and CMX samples are slightly different.
- $\epsilon_{evt}$  is the total pretag selection acceptance including all relevant scale factors that take into account the discrepancies between efficiencies calculated on data and on Monte Carlo. In table 5.1 we show the most important ones.
- $\epsilon_{tag}$  is the tagging efficiency that it is equal to one for the pretag sample.

After the calculation of the electroweak processes normalization, we measure the QCD fraction in our sample by performing a likelihood fit to the data on the  $\cancel{E}_T$  distribution using a non-W template and a signal template.

In the pretag sample events that are not QCD, electroweak or top are assumed to be W+jets. Normalization of this background is obtained by:

$$N_{W+Jets}^{pretag} = N_{pretag} \cdot (1 - F_{QCD}^{pretag}) - N_{ewk}^{pretag} - N_{top}^{pretag} \quad (4)$$

Our sample is divided into 18 subsamples, organized by type of leptons (CEM, CMUP or CMX), by number of jets in the event (3,4,  $\geq 5$ ) and by number of b-tagged jets (1 or 2). In the following we show the distribution of a set of variables in data and in Monte Carlo simulation for the various leptons (CEM+CMUP+CMX). In Table 1 and in Table 2 we list, respectively for 1-Tag or 2-Tags in the final state the expected number of events. These results are obtained using  $R = 1$  and  $\sigma_{p\bar{p} \rightarrow t\bar{t}} = 7.04 \pm 0.49$  pb.

Lepton+Jets, 1b Tag	CDF Preliminary 8.7fb <sup>-1</sup>		
	3 Jets	4 Jets	$\geq 5$ Jets
$t\bar{t}$	800 ± 67	777 ± 64	260 ± 21
S <sub>TopS</sub>	30 ± 2	6.5 ± 0.5	1.5 ± 0.1
S <sub>TopT</sub>	48 ± 5	10 ± 1	2.1 ± 0.2
WW	33 ± 4	8 ± 1	2.4 ± 0.3
WZ	9.9 ± 0.9	1.9 ± 0.2	0.6 ± 0.1
ZZ	1.8 ± 0.2	0.46 ± 0.04	0.12 ± 0.01
Z+jets	31 ± 3	9.1 ± 0.9	2 ± 0.2
W+bb	291 ± 118	74 ± 30	17 ± 7
W+cc	167 ± 68	47 ± 20	12 ± 5
W+c	87 ± 35	17 ± 7	4 ± 2
Mistags	303 ± 42	74 ± 14	17 ± 6
Non-W	125 ± 50	35 ± 29	10 ± 9
Total Prediction	1928 ± 243	1061 ± 93	330 ± 28
Observed	1844	1088	339

Table 1: Estimates in the 1 tag bin for the yields of the different contributions collected by all detectors

## VI. $R$ dependent signal and background estimates

The signal and background modeling described so far is performed under the Standard Model assumption of  $R = 1$ . This assumption affects also the value of the  $\sigma_{t\bar{t}}$  used for the top pair events normalization. The most important effect due to a  $R \neq 1$  is on the number of  $b$ -tagged events: the smaller  $R$ , the lesser is the probability to have a  $b$  jet in the top pair events. Therefore events with one or two tags are expected to decrease with decreasing  $R$ .

The expected number of  $t\bar{t}$  events in the tag subsamples, using equation 3 is given by:

Lepton+Jets, 2b Tag		CDF Preliminary 8.7fb <sup>-1</sup>	
Process	3 Jets	4 Jets	≥5 Jets
$t\bar{t}$	216 ± 30	271 ± 36	97 ± 13
STopS	9 ± 1	2.0 ± 0.3	0.47 ± 0.06
STopT	9 ± 1	2.7 ± 0.4	0.60 ± 0.09
WW	0.9 ± 0.2	0.5 ± 0.1	0.22 ± 0.04
WZ	1.8 ± 0.3	0.27 ± 0.04	0.07 ± 0.01
ZZ	0.4 ± 0.1	0.14 ± 0.02	0.01 ± 0.00
Z+jets	2.1 ± 0.2	0.77 ± 0.08	0.29 ± 0.03
W+bb	48 ± 20	14 ± 6	4 ± 2
W+cc	5 ± 2	2 ± 1	0.8 ± 0.4
W+c	3 ± 1	0.8 ± 0.4	0.2 ± 0.1
Mistags	5 ± 1	1.7 ± 0.4	0.6 ± 0.2
Non-W	6 ± 3	0.1 ± 1.5	0.1 ± 1.5
Total Prediction	306 ± 40	296 ± 38	104 ± 13
Observed	275	273	126

Table 2: Estimates in the 2 tag bin for the yields of the different contributions collected by all detectors

$$\mu_{t\bar{t}}^{i,j}(R) = \mathcal{L}^j \cdot \epsilon_{evt}^{i,j} \cdot \sigma_{t\bar{t}} \cdot \epsilon_{tag}^i(R) \quad (5)$$

where  $i$  and  $j$  indicate respectively the  $i$ -th jet bin (3,4, ≥ 5) and  $j$ -th considered detector (CEM, CMUP or CMX). In the formula 5,  $\mathcal{L}^j$  is the integrated luminosity for each type of detector,  $\epsilon_{evt}^{i,j}$  is the event detection efficiency and  $\epsilon_{tag}^i(R)$  is the event tag efficiency, calculated in detail in next section.

The background estimates are expected to be almost independent upon  $R$ , except for the single top production that is affected by the top quark decay ratio. We decided to neglect this effect since the single top yields are ≈ 9% of the top pair events in the 3 jet bin, ≈ 2% in the 4 jet bin and ≈ 1% in the 5 jet bin.

The event tagging efficiencies are crucial for the determination of the number of top pair events in each tag bin since they determine the fraction of  $t\bar{t}$  events with zero, one or two  $b$  tagged jets. These efficiencies are calculated in MC, using the *SecVtx* algorithm, taking into account the jet tagging efficiencies for  $b$  and  $c$  jets and data/MC differences (i.e. correcting for a Scale Factor). Probability to (mis)tag a light flavour jet is obtained using a mistag matrix which parameterizes this probability as a function of jet characteristics.

In general,  $\epsilon_{tag}$  is calculated from the probability  $P_{tag}^{event}$  to tag an event with a given number of jets. The general formulas for the  $P_{tag}^{event}$  for a generic number of jets  $n$  in the event are:

$$P_{1-tag}^{event} = \sum_{i=1}^n p_i^{tag} \cdot \left( \prod_{j=1, j \neq i}^n (1 - p_j^{tag}) \right) \quad (6)$$

$$P_{2-tag}^{event} = \sum_{i=1}^{n-1} p_i^{tag} \cdot \left( \sum_{j>i}^n p_j^{tag} \cdot \left( \prod_{k=1, k \neq i, k \neq j}^n (1 - p_k^{tag}) \right) \right) \quad (7)$$

where  $p_i^{tag}$  is the probability to tag the  $i$ -th jet in the event. For jets matched<sup>1</sup> to heavy flavor, either  $b$  or  $c$  jets,  $p_i^{tag}$  is the tagging scale factor SF if the jet is tagged. If the jet is matched to a light flavor  $p_i^{tag}$  is the mistag probability, calculated by the mistag matrix.

The  $P_{tag}^{event}$  is the weight given to each event, used to calculate the tagging efficiency to have  $k$  tags by:

<sup>1</sup>Here *matched* means that a heavy flavor hadron generated by the Monte Carlo showering has been found inside the jet cone. This does not imply that the jet can be efficiently tagged by *SecVtx*.

$$\epsilon_{k-tag} = \frac{\sum_j^{events} P_{k-tag}^j}{N_{pretag}}. \quad (8)$$

where the sum run over all pretagged events. This procedure to calculate the tagging efficiency is of general application and is used by our algorithm to calculate the number of events in the tag samples. The larger source of uncertainty on  $\epsilon_{k-tag}$  is due to the b-tagging algorithm. Its effect is estimated by shifting the SF and the mistag matrix by  $\pm 1\sigma$  w.r.t.the central value, and by applying again the whole algorithm. Other sources of systematic uncertainties on  $\epsilon_{k-tag}$  are the Jet Energy Scale and signal modeling. We performed the calculation with the JES shifted by  $\pm 1\sigma$  with respect to the central value, while we studied the impact of the signal modeling using a different Monte Carlo generator (HERWIG[6]) for the ditop events as described in Section 10.

The Monte Carlo sample used for the signal modeling is generated using the CKM matrix element  $|V_{tb}| = 1$  so it can not be used directly to calculate the  $\epsilon_{tag}$  as a function of  $R$  through the algorithm described above. For the calculation of the event tag efficiency we evaluated the correct weight for each event in the following way:

- For every event we select the jets matched to a  $b$  quark coming from a  $t$  quark at parton level.
- For every jet matched in this way we extract a random number  $P_b$  in the interval  $[0,1]$ .
- If  $P_b < R$  we consider this jet as a real  $b$  quark, otherwise this jet is considered as a LF-jet.
- We apply the ordinary sample composition estimate logic to the  $p_i^{tag}$ .

In the formulas above the probability to tag a jet is simply the tag Scale Factor defined by  $SF = \frac{\epsilon_{tag}^{data}}{\epsilon_{tag}^{MC}} = 0.96 \pm 0.05$  for jets matched to a heavy flavor quark, while, if the jet is matched to a light flavor,  $p_i^{tag}$  is calculated building the *mistag-matrix* for the event.

In this way we are considering that the  $b$  quark produced in the top decay is a real  $b$  only  $R$  times while  $(1 - R)$  times it is considered a light flavor quark and it is weighted by the mistag probability. The formula reproduces exactly the standard calculation in the case of  $R = 1$ , simulates  $t \rightarrow Wq$  for  $R=0$  and allows us to calculate  $\epsilon(R)$  trough Eq. 8 in each tag subsample and in each jet bin.

Background and signal are then calculated from the sample composition estimate for various  $R$ .

In Table 3 we show, as example of the calculation, the results for  $R = 0.1$ ,  $R = 0.5$ , both for the total and the background estimates, for the events collected by the three detectors. In Figure 1 we show the data collected compared to the total prediction for different  $R$  values for the possible final states.

The method described above let us to model the number of  $t\bar{t}$  events, and consequently the background counts, as a function of  $R$  in a straightforward way.

		CDF Preliminary 8.7fb <sup>-1</sup>					
		1b Tag			2b Tags		
		3jets	4jets	≥5jets	3jets	4jets	≥5jets
	R=0.1	1117 ± 230	279 ± 65	68 ± 18	88 ± 24	25 ± 8	7 ± 3
		1365 ± 232	561 ± 72	186 ± 22	100 ± 24	46 ± 9	19 ± 4
Background ( $N_B^i$ )	R=0.5	1123 ± 232	282 ± 68	70 ± 18	90 ± 27	25 ± 9	7 ± 3
	Total Expected ( $\mu_{exp}^i$ )	1680 ± 238	857 ± 84	270 ± 25	162 ± 27	124 ± 17	47 ± 7
	R=1.0	1127 ± 233	283 ± 67	69 ± 18	90 ± 27	25 ± 11	7 ± 3
		1927 ± 243	1060 ± 93	330 ± 28	306 ± 40	296 ± 38	104 ± 13
Observed( $N_{obs}^i$ )		1844	1088	339	275	273	126

Table 3: Predicted and observed number of events with at least three jets in the final state. The background and total estimates are shown separately according to different values of  $R$  (in each row background is the value on the top).

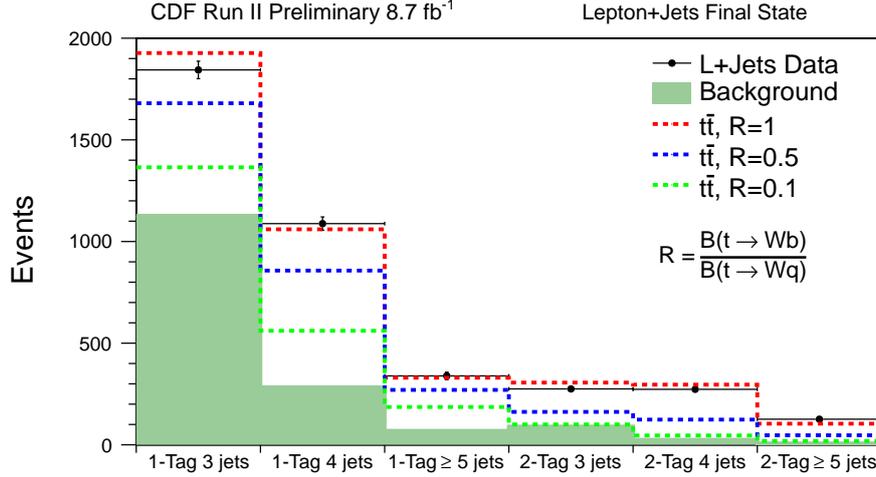


Figure 1: Data collected compared to the total expected events for different values of  $R$  as a function of different final states. For the  $t\bar{t}$  normalization we used  $\sigma_{p\bar{p} \rightarrow t\bar{t}} = 7.04 \pm 0.49$  pb.

## VII. Likelihood Function

We have 18 subsamples in total where we estimate the  $t\bar{t}$  and background processes content. In order to compare the prediction to the observed data we use a Likelihood function. Our procedure simultaneously fits, by minimizing the negative logarithm of the Likelihood,  $R$  and  $\sigma_{p\bar{p} \rightarrow t\bar{t}}$ .

The likelihood function used for the analysis is :

$$\mathcal{L} = \prod_i \mathcal{P}(\mu_{exp}^i(R, \sigma_{p\bar{p} \rightarrow t\bar{t}}, x_j) | N_{obs}^i) \prod_j G(x_j | 0, 1) \quad (9)$$

In this expression,  $\mathcal{P}(\mu_{exp}^i(R, x_j) | N_{obs}^i)$  is the Poisson probability to observe  $N_{obs}$  in the  $i$ -th bin, given the expected mean  $\mu_{exp}$ . The index  $i$  runs on all 18 bins. The functions  $G(x_j | 0, 1)$  are normals in the variable  $x_j$  and are used to model sources of systematic uncertainties. We first compute the central value for each parameter  $\bar{A}$  and its uncertainty  $\sigma_A^k$  relative to a single source of systematics. We define:

$$A(x_k) = \bar{A} + \sum_k^{syst} x_k \cdot \sigma_A^k \quad (10)$$

For discrete<sup>2</sup> systematics we computed the shift of the central value for an increment or for a decrement of the systematic  $k$  defining:

$$\sigma_{+A}^k = |\bar{A}(+1\sigma^k) - \bar{A}| \quad (11)$$

$$\sigma_{-A}^k = |\bar{A}(-1\sigma^k) - \bar{A}| \quad (12)$$

And then symmetrizing to

$$\sigma_A^k = \frac{\sigma_{+A}^k + \sigma_{-A}^k}{2} \quad (13)$$

in order to use Equation 10. This procedure correlates uncertainties among channels by using the same parameter for a common source of systematic uncertainty and allows each parameter to vary with respect to its central value. In addition the parameters uncertainty can be statistic, systematic or a combination of both. In the Poisson function the expected mean  $\mu^i(R, x_j)$  is given by the total prediction

<sup>2</sup>For *discrete* we mean a systematic that can be switched in this analysis only by  $+1\sigma$  or  $-1\sigma$  in a discrete manner, *i.e.* the Jet Energy Scale

Lepton+Jets CDF Preliminary 8.7 fb <sup>-1</sup>	
$\sigma_{p\bar{p}\rightarrow t\bar{t}}$ (pb)	$7.5 \pm 0.3$ (stat) $\pm 0.9$ (syst)
R	$0.94 \pm 0.04$ (stat) $\pm 0.09$ (syst)

Table 4: Fit results for for  $R$  and  $\sigma_{t\bar{t}}$ .

extracted from the calculation described in the previous section and it is the sum of the expected  $t\bar{t}$  yields and the background estimates  $N_B^i$ .

In order to fit the parameters to the observed data we performed the minimization of the  $-2\log(\mathcal{L})$ , using the MINUIT package. From the negative likelihood logarithm minimization, we obtain two results for the free parameters ( $R$  and  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$  in our case). The uncertainty on a parameter is calculated in the following way. We follow the function out from the minimum, finding the crossing point with the function value  $min + 1$ . This method in general provides different positive and negative errors and the difference between the symmetric and asymmetric uncertainties is a measure of the non-linearity of the model. The uncertainties obtained in this way are usually larger than uncertainties derived from the error matrix.

### VIII. Fit on Data

Finally we looked at the lepton plus jets events in the data. The starting point of our fit is the expectation for signal and background, obtained using  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$  from the kinematics measurement [13]  $\sigma_{t\bar{t}} = 7.82 \pm 0.55$  pb. In order to avoid any further bias, we let  $R$  and  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$  vary and we performed a recursive procedure that we describe below:

- first we performed a simultaneous fit on both  $\sigma_{t\bar{t}}$  and  $R$ , by minimizing the negative logarithm of the likelihood function;
- we ran again the background calculation using the new  $\sigma_{t\bar{t}}$  returned from the fit;
- we iterated the previous steps until the procedure converged.<sup>3</sup>

As a check we performed the recursive fit also starting from the theoretical top cross section  $\sigma_{p\bar{p}\rightarrow t\bar{t}} = 7.04 \pm 0.49$  (pb). The fit converged to the same results. Moreover we performed a pure statistical fit, fixing all systematics to their nominal value, and a combined statistical plus systematics fit, where the single sources of systematics are let free to fluctuate about their mean as described in previous section. In the following plots we show the shape of  $-2\Delta\log(\mathcal{L}(R, \sigma_{p\bar{p}\rightarrow t\bar{t}}))$ , defined as:

$$-2\Delta\log(\mathcal{L}(R, \sigma_{p\bar{p}\rightarrow t\bar{t}})) = -2 \cdot (\log(\mathcal{L}(R, \sigma_{p\bar{p}\rightarrow t\bar{t}})) - \log(\mathcal{L}(R^{min}, \sigma_{p\bar{p}\rightarrow t\bar{t}}^{min}))) \quad (14)$$

where the superscript *min* indicates the values at the absolute minimum. In this way we shift the Likelihood value at the minimum from zero. For a better understanding of the behaviour we also show the Likelihood Fit projected onto the  $R$  and  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$  axes respectively, for both the statistical and the combined fit. There is no indication of other minima in a wide range around the minimum found. We extract the  $\pm 1\sigma$  uncertainties crossing  $y = -2\Delta\log(\mathcal{L}(R, \sigma_{p\bar{p}\rightarrow t\bar{t}})) = 1$  is indicated on the figure by a red dotted line. We can observe that the central values for  $R$  does not change under the two different fits, while the fit value for  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$  changes only slightly.

<sup>3</sup>For convergence we mean here that the difference between the  $n$ -th fitted  $R$  value and the  $(n - 1)$ -th one is less than 1/10 of the uncertainty on the parameter.

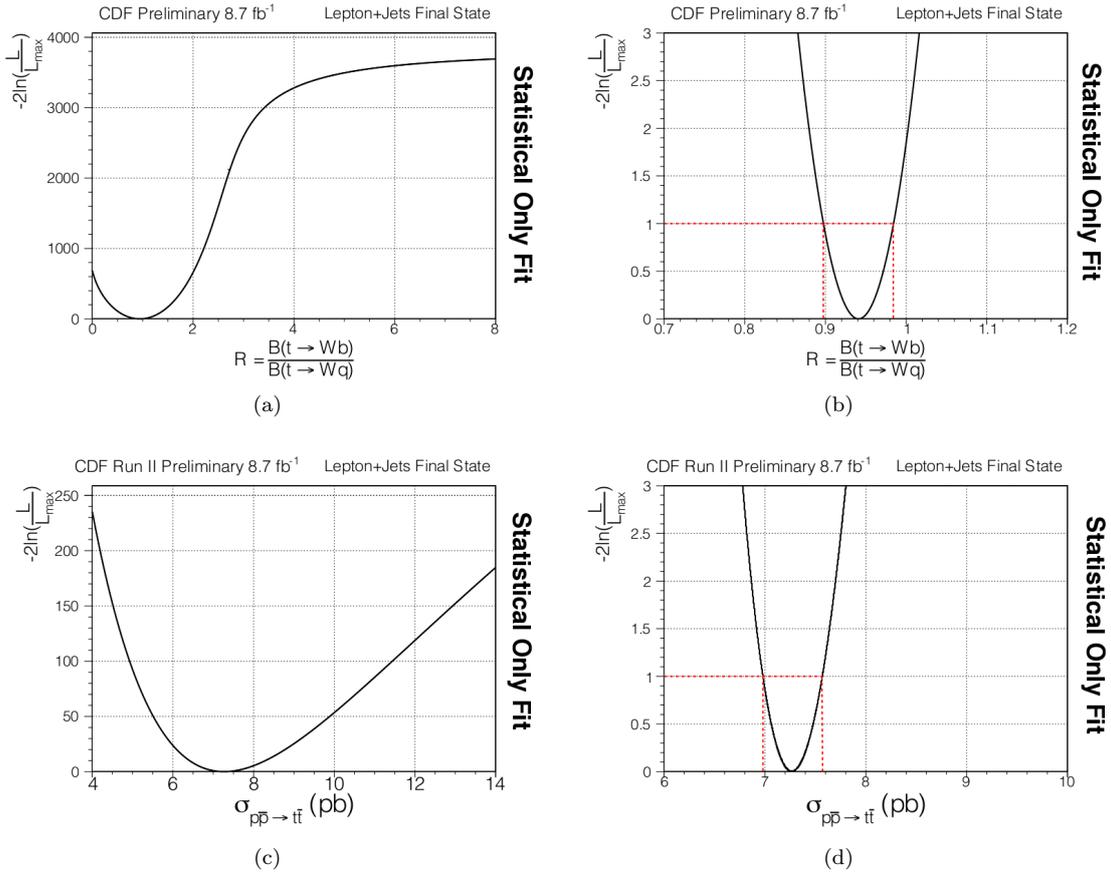


Figure 2: Pure statistical fit. (a) Plot of  $-2 \cdot \ln(\mathcal{L}(R))$  in a wide range and (b) a zoom in the minimum region. (c) Plot of  $-2 \cdot \ln(\mathcal{L}(\sigma_{p\bar{p} \rightarrow t\bar{t}}))$  in a wide range and (d) a zoom in the minimum region.

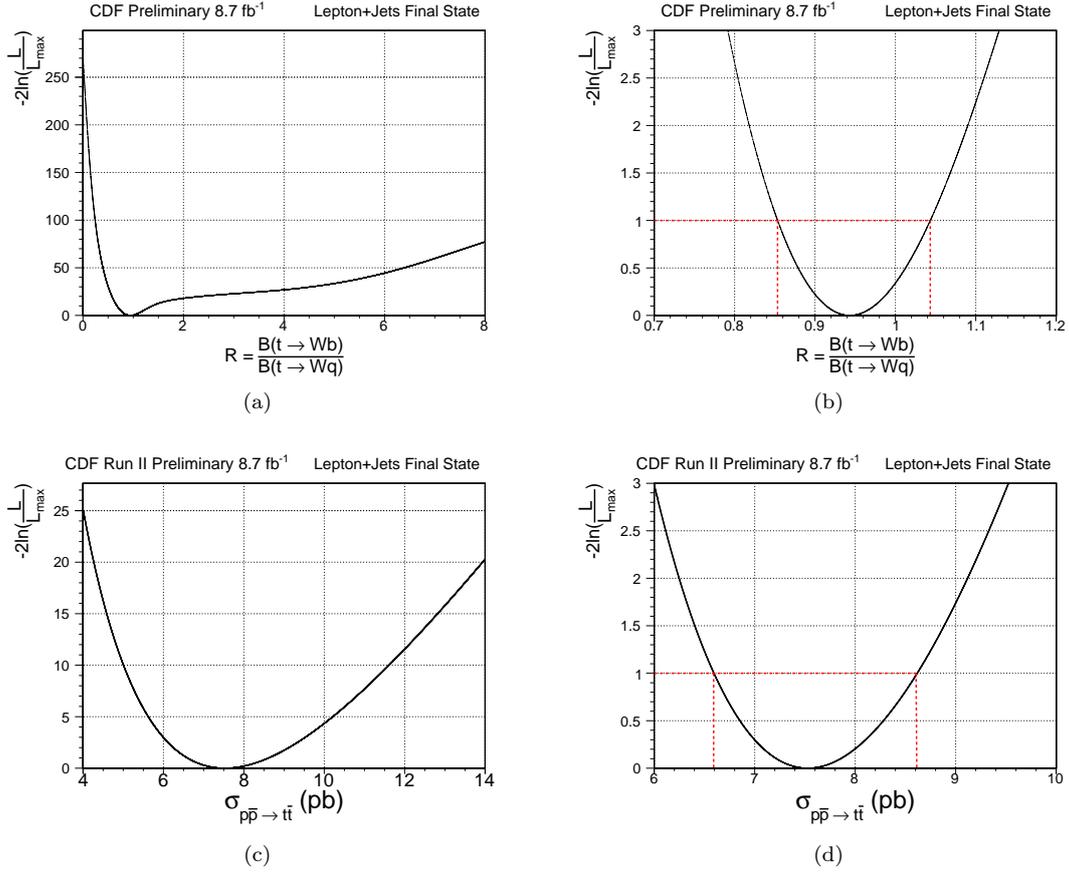


Figure 3: Combined statistical plus systematic fit. (a) Plot of  $-2 \cdot \ln(\mathcal{L}(R))$  in a wide range and (b) a zoom in the minimum region. (c) Plot of  $-2 \cdot \ln(\mathcal{L}(\sigma_{p\bar{p} \rightarrow t\bar{t}}))$  in a wide range and (d) a zoom in the minimum region.

## IX. Systematics

The uncertainty given by the fit is comprehensive of the systematics on event tagging efficiency, due to the combination of the tagging scale factor and the mistag matrix, the event selection efficiency, including the lepton identification scale factor and the trigger efficiency, the  $z_0$  cut systematic, the background normalizations, including the heavy flavor fractions, corrections for MC-data heavy flavour yield and the luminosity. We think that the procedure of adding both statistical and systematic errors in the Likelihood returns a good estimate of the total uncertainty, since we are dealing with many sources of uncertainty at the same time. We also included the contributions due to the Jet Energy Scale, ISR/FSR, signal modeling and Top Quark Mass. This set of systematics is folded to the Likelihood through nuisance parameters and affects the event tagging efficiencies, background normalizations and acceptancies and top pair selection efficiencies. In order to compute properly the impact of this set of systematics we used different Monte Carlo samples to simulate signal or backgrounds.

- **Jet Energy Scale (JES):** The impact of the JES uncertainty is estimated by varying the energy of all jets in the Monte Carlo samples by  $\pm 1\sigma_{JES}$  for both signal and backgrounds. This produces a variation of the acceptances, of the event tag efficiencies and a mixing of the number of events between various bins.
- **Process generator:** we use PYTHIA. In order to estimate the possible bias introduced by its use, we performed a new analysis using for the  $t\bar{t}$  signal a sample with the matrix element generated by HERWIG and the showering performed by PYTHIA.
- **Initial and Final State Radiation (ISR/FSR):** in order to estimate the effect of this systematic on our measurement we used two different Monte Carlo samples for  $t\bar{t}$  signal where the ISR/FSR is respectively enhanced or reduced.
- **Top quark mass ( $m_t$ ):** the top quark production cross section depends on  $m_t$  [14, 15] and is experimentally measured [16]. Since in this analysis we perform a recursive fit on the  $\sigma_{t\bar{t}}$  we expect to reduce the impact of this systematic, but, in order to check this assumption, we performed a new measurement using two different Monte Carlo samples for the  $t\bar{t}$  signal respectively at  $m_t = 170$  GeV/ $c^2$  and  $m_t = 175$  GeV/ $c^2$ .

After obtaining central values and uncertainties on those systematics, they are included in the Likelihood as nuisance parameters.

The uncertainty returned by the combined statistical plus systematic fit, is the total uncertainty on our parameter of interest. The effect of each single source of systematic is calculated via pseudo experiments. We generated a set of pseudo experiments with the same prescription but with the nuisance parameter  $x_k$ , relative to the systematic under study, shifted by one standard deviation from its mean and we calculated the shift induced on mean of the distributions for  $R$  and  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$ . To obtain the total uncertainty on  $R$  and  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$ , these contributions were summed in quadrature. The result can be slightly different from the one obtained with the global fit.

## X. Results

The final results obtained with  $8.7 \text{ fb}^{-1}$  of data, using the *SecVtx* tagged lepton plus jets sample are summarized in Table 7. In Figure 4 we show the bidimensional contours by the combined fit. The fitted value, with its one dimensional uncertainties, is marked in the plot with a cross and can be compared to the theoretical Standard Model prediction at NLO [4]. The results are in agreement with the theoretical prediction within  $1\sigma$ .

To determine a lower limit at some confidence level on  $R$  we follow a Bayesian statistical approach. Since  $R$  is bounded to be in the interval  $[0,1]$ , the prior  $\pi(R)$  is chosen to be zero outside this  $R$  boundaries while we consider all physical values equally probable. The prior is given by

Lepton + Jets		CDF Preliminary 8.7 fb <sup>-1</sup>	
Source	+ $\delta R$	- $\delta R$	
Statistical	0.043	-0.043	
Jet Energy Scale	0.016	-0.019	
ISR/FSR	0.006	-0.006	
$b$ -tagging	0.078	-0.073	
Background Normalization	0.056	-0.052	
Others	0.005	-0.005	
Squared Sum	0.098	-0.092	

Table 5: Major uncertainties on the measurement of  $R$  divided for each class of systematic source. With the source “Others” we mean the squared sum of minor systematics.

Lepton + Jets		CDF Preliminary 8.7 fb <sup>-1</sup>	
Source	+ $\delta\sigma_{p\bar{p}\rightarrow t\bar{t}}$ (pb)	- $\delta\sigma_{p\bar{p}\rightarrow t\bar{t}}$ (pb)	
Statistical	0.29	-0.29	
Jet Energy Scale	0.46	-0.41	
ISR/FSR	0.22	-0.21	
Luminosity	0.44	-0.39	
Background Normalization	0.78	-0.66	
Top Mass	0.33	-0.32	
Others	0.18	-0.15	
Squared Sum	1.08	-0.96	

Table 6: Major uncertainties on the measurement of  $\sigma_{p\bar{p}\rightarrow t\bar{t}}$  divided for each class of systematic source. With the source “Others” we mean the squared sum of minor systematics.

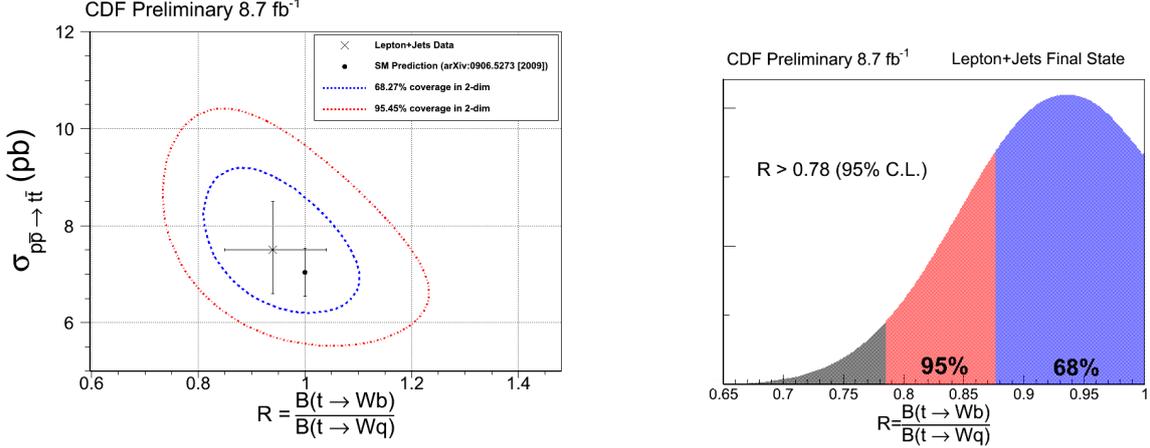


Figure 4: Left: Bidimensional contours for the combined statistical plus systematic fit. Colors and labels online. Right: Lower limits on  $R$  at various confidence levels.

$$\pi(R) = \begin{cases} 1, & \text{if } 0 \leq R \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

To establish a lower limit we calculate the following integral

$$P_{R \geq \bar{R}} = \frac{\int_{\bar{R}}^1 \mathcal{L}(R, \vec{x}) dR d\vec{x}}{\int_0^1 \mathcal{L}(R, \vec{x}) dR d\vec{x}} \quad (15)$$

where the value of  $P_{R \geq \bar{R}} = 0.68, 0.95, \dots$  gives us the requested confidence level. The marginalization of the Likelihood function with respect to the other parameters has been performed using the saddle point approximation [17], that leads to

$$\mathcal{L}^*(R) = \int d\vec{x} \mathcal{L}(R, \vec{x}) = \mathcal{L}_{max}(R) \sqrt{\det(C(\vec{x}))} \quad (16)$$

where  $C(\vec{x})$  is the covariance matrix of the  $\vec{x}$  parameters. The marginalized likelihood was obtained by a numerical integration and then fitted with a sum of two bifurcated gaussians with same mean and different widths. The fit is in perfect agreement with the distribution as shown in Figure ?? (a). Using Equation 15 we obtain the Bayesian lower limits at 68% CL and 95% CL and the results are shown in Figure ?? (b). We measure  $R > 0.785$  at 95% C.L. From Equation 1 we extract a measurement of  $V_{tb}$ . Assuming three generation of quarks and the unitarity of the CKM matrix, we have  $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$ , leading to  $R = |V_{tb}|^2$ . From our fit results we obtain  $|V_{tb}| = 0.97 \pm 0.05$  and  $|V_{tb}| > 0.89$  at 95% C.L.

Lepton + Jets	CDF Preliminary 8.7fb <sup>-1</sup>		
	Value (stat+syst)	Lower Limit 68% CL	Lower Limit 95% CL
$\sigma_{p\bar{p} \rightarrow t\bar{t}}$ (pb)	$7.5 \pm 1.0$	-	-
R	$0.94 \pm 0.09$	0.876	0.785
$ V_{tb} $	$0.97 \pm 0.05$	0.936	0.886

Table 7: Analysis results showing the measured values and the relative lower limits at various confidence levels

## XI. Conclusions

In this analysis we updated the CDF measurement of the top branching fraction ratio  $R = \frac{\mathcal{B}(t \rightarrow Wb)}{\mathcal{B}(t \rightarrow Wq)} = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2}$  in the  $l+$  jets channel, combined with a measurement of the top pair production cross section  $\sigma_{p\bar{p} \rightarrow t\bar{t}}$ . We measure  $R = 0.94 \pm 0.1(\text{stat+syst})$  and  $\sigma_{t\bar{t}} = 7.5 \pm 0.95$  pb. This result is dominated by the systematic uncertainty, in contrast with the old CDF measurements.

Results for  $\sigma_{p\bar{p} \rightarrow t\bar{t}}$ ,  $R$  and  $|V_{tb}|$  are in agreement with the Standard Model, with the previous CDF measurements and with the latest measurement of  $R$  performed by DØ [3].

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