

A measurement of $\sin^2\theta_{\text{eff}}^{\text{lept}}(M_Z)$, $\sin^2\theta_w$ and indirect measurement of M_w at CDF

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On behalf of the CDF Collaboration

ICHEP 2014:

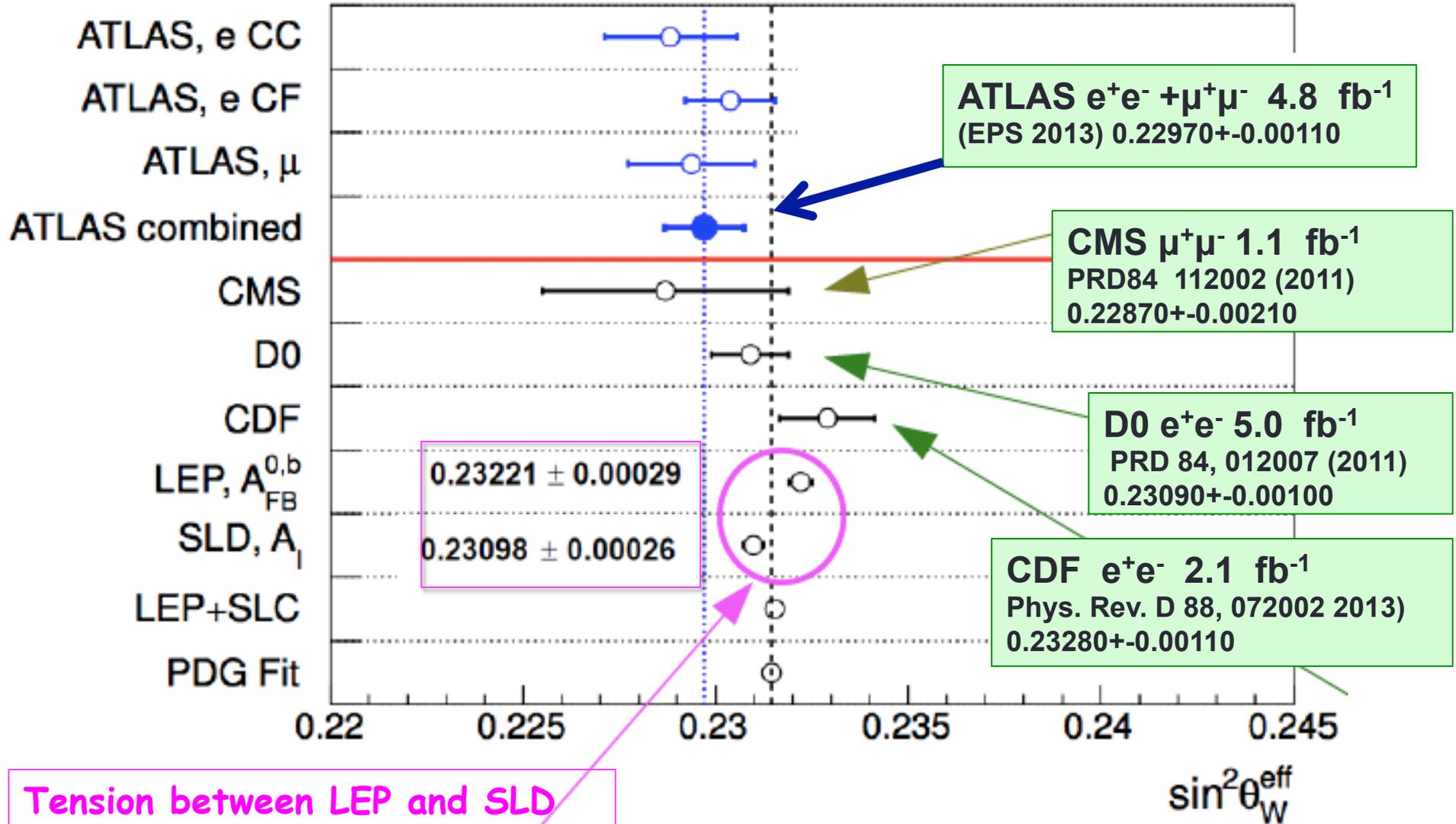
Saturday, July 5, 2014 9:00-9:20 (17+3). Room: Sala 8+9

Session: Top-quark and Electro Weak Physics

Ref: [CDF, Phys. Rev D. 89, 072005 \(2014\)](#)
[arXiv:1402.2239v2](#)



2013: Long standing tension between LEP/SLD values of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ (Mz)



Tension between LEP and SLD

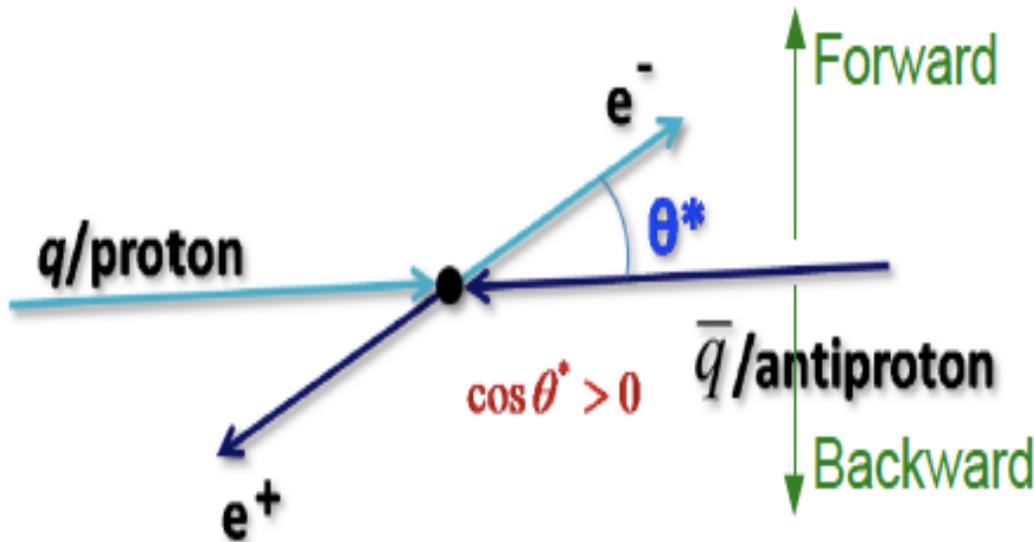
LEP SLD difference is 0.00122 New precision measurements of $\sin^2\theta_{\text{eff}}$ would help resolve this diff.

$\sin^2\theta_w = 1 - M_w^2 / M_z^2$ Is independent of mass or lepton or quark flavor

In hadron colliders: A_{FB} for e^+e^- or $\mu^+\mu^-$ pairs in the Z boson Region is sensitive to $\sin^2\theta_{\text{eff}}^{\text{lept}}(M_{\mu+\mu})$ (which depends on mass and quark flavor)

Born level polar angle distribution: $1 + \cos^2\theta + A_4 \cos\theta$

Collinear, No dilepton PT



$$A_{FB} = (3/8) A_4$$

Define Forward-Backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

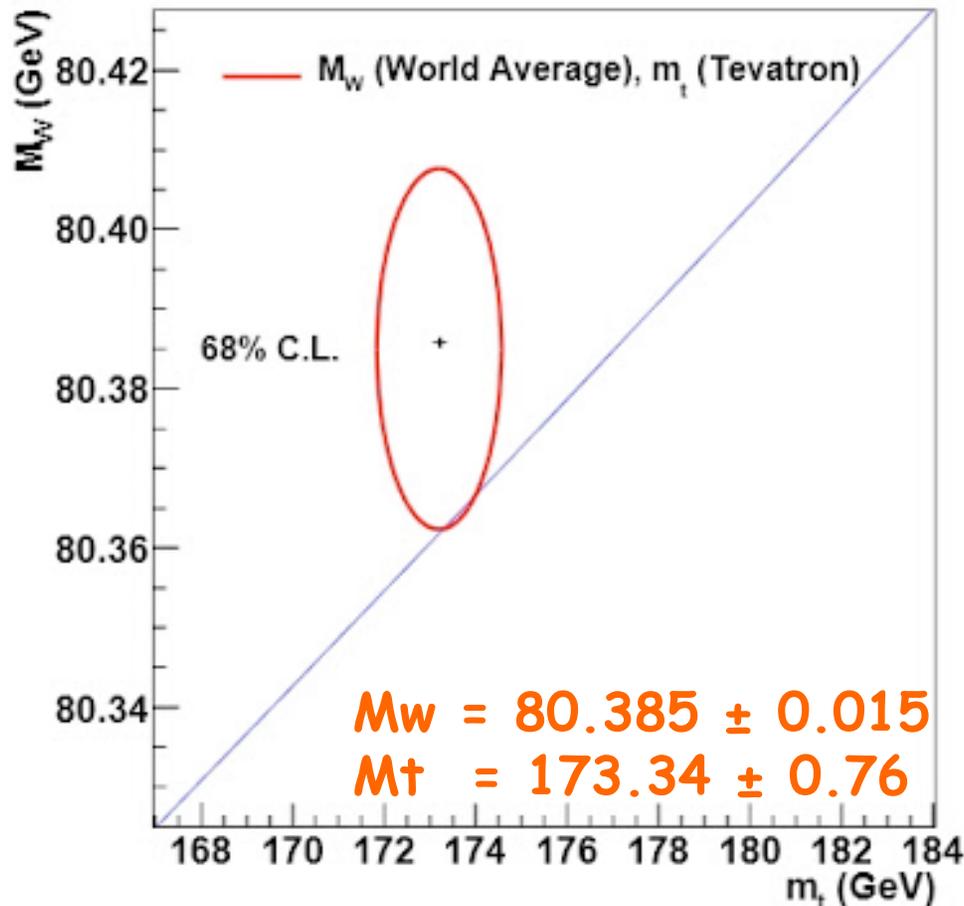
We define for short: $\sin^2\theta_{\text{eff}} \equiv \sin^2\theta_{\text{eff}}^{\text{lept}}(M_z)$ **at the Z pole!**

The following is an approximate relation

$$\sin^2\theta_{\text{eff}}^{\text{lept}} \approx 1.037 \cdot \sin^2\theta_w \quad [\text{ZFITTER } \kappa_e(\sin^2\theta_w, M_z) \text{ form factor}]$$

Direct and indirect measurements of M_W in SM

The new key element in the indirect extraction or inference of M_W from A_{FB} in the Standard Model is that the Higgs mass is now known. Therefore we can measure both $\sin^2\theta_{\text{eff}}$ AND the on-shell $\sin^2\theta_w = 1 - M_W^2 / M_Z^2$ (we use $m_H = 125 \text{ GeV}$).

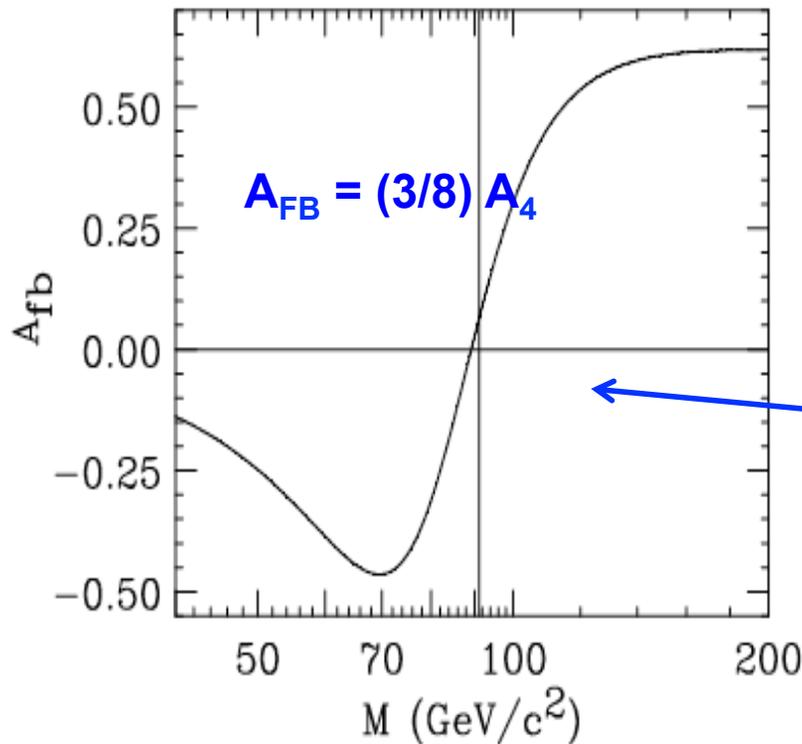


An indirect measurement of M_W is done by measuring the on-shell $\sin^2\theta_w$ and using the SM relation

$$\sin^2\theta_w = 1 - M_W^2 / M_Z^2$$

A error of ± 0.00030 in $\sin^2\theta_w$ is equivalent to an indirect measurement of M_W to a precision of $\pm 15 \text{ MeV}$

W mass provides a stringent test of the SM. Within SM we can measure the W mass both directly and indirectly. They should agree.



For finite dilepton PT

$$\frac{dN}{d\Omega} \propto (1 + \cos^2 \vartheta) +$$

$$A_0 \frac{1}{2} (1 - 3 \cos^2 \vartheta) +$$

$$A_1 \sin 2\vartheta \cos \varphi +$$

$$A_2 \frac{1}{2} \sin^2 \vartheta \cos 2\varphi +$$

$$A_3 \sin \vartheta \cos \varphi +$$

$$A_4 \cos \vartheta +$$

$$A_5 \sin^2 \vartheta \sin 2\varphi +$$

$$A_6 \sin 2\vartheta \sin \varphi +$$

$$A_7 \sin \vartheta \sin \varphi .$$

Terms in boxes are zero when integrating over φ , and we get

$$\left(1 + \cos^2 \theta \right) + A_0(M, P_T) \frac{(1 - 3 \cos^2 \theta)}{2} + A_4 \cos \theta$$

Note that A_{FB} is not Zero at the Z pole. Most of the sensitivity to $\sin^2 \theta_{eff}$ is at the Z pole.

For dileptons with a P_T , the change in the $\cos \theta$ distribution in the Collins-Soper frame is well understood. CDF has measured it, and data agrees with POWHEG QCD prediction. It is accounted for in the analysis and does not have much impact to the results.

We report on the following CDF measurement published in 2014.
Phys. Rev D. 89, 072005 (2014) full Run II data set $9 \text{ fb}^{-1} \mu^+\mu^-$

CDF analysis uses three new innovations which are essential:

1st innovation: $\sin^2\theta_W$ is constant $\rightarrow \sin^2\theta_{\text{eff}}^{\text{lept}}$ (M_Z , flavor)

Full ZFITTER EW radiative corrections, Enhanced Born Approximation (EBA), include full complex form factors

(implemented private versions of RESBOS, POWHEG, and LO)

CDF: Phys. Rev. D 88, 072002 (2013) Appendix A'

[arXiv:1307.0770v3](https://arxiv.org/abs/1307.0770v3) [hep-ex]

2nd innovation:

Precise lepton momentum/energy scale corrections using a new method

A. Bodek et al. Euro. Phys. J. C72, 2194 (2012)

[arXiv:1208.3710v3](https://arxiv.org/abs/1208.3710v3) [hep-ex]

3rd innovation: Event weighting method for A_{FB} analyses

(all systematic errors in acceptance and efficiencies cancel)

A. Bodek. Euro. Phys. J. C67, 321 (2010)

[arXiv:0911.2850v4](https://arxiv.org/abs/0911.2850v4) [hep-ex]

1st innovation: $\sin^2\theta_W$ is constant $\rightarrow \sin^2\theta_{\text{eff}}^{\text{lept}} (M_Z, \text{flavor})$
Full FITTER EW radiative corrections Enhanced Born Approximation (EBA)

Implemented by the Rochester CDF group (**Willis Sakumoto**, A. Bodek, J.-Y. Han),
 see Phys. Rev. D88, 072002 (2013) Appendix A [arXiv:1307.0770v3](https://arxiv.org/abs/1307.0770v3) [hep-ex]

$g_V^f \gamma_\mu + g_A^f \gamma_\mu \gamma_5$. The Born-level couplings are

$$g_V^f = T_3^f - 2Q_f \sin^2 \theta_W$$

$$g_A^f = T_3^f,$$

If RESBOS is used then the EBA EW correction to $\sin^2\theta_{\text{eff}} = 0.00031 \pm 0.00012$
 Vs. stat error 0.00080 ($\mu^+\mu^-$) **9 fb⁻¹**
 Vs. stat error 0.00040 (e^+e^-) **9 fb⁻¹**

They are modified by ZFITTER 6.43 form factors (which are complex)

$$g_V^f \rightarrow \sqrt{\rho_{\text{eq}}} (T_3^f - 2Q_f \kappa_f \sin^2 \theta_W), \quad \text{and} \quad \text{SM}(\sin^2 \theta_W) \xrightarrow{\text{EWK}} \sin^2 \theta_{\text{eff}}(s) \xleftrightarrow{\text{QCD}} A_4(s),$$

$$g_A^f \rightarrow \sqrt{\rho_{\text{eq}}} T_3^f, \quad A_{\text{FB}} = (3/8) A_4$$

- T_3 and $\sin^2\theta_W \rightarrow$ **effective T_3 and $\sin^2\theta_W$** : 1-4% multiplicative form factors
- On-mass shell scheme: $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$ to all orders

$$\sin^2\theta_{\text{eff}}^{\text{lept}} \simeq 1.037 \cdot \sin^2\theta_W \quad [\text{ZFITTER } \kappa_e(\sin^2\theta_W, M_Z) \text{ form factor}]$$

2nd innovation: **Precise momentum/energy scale corrections**

A. Bodek et al. Euro. Phys. J. C72, 2194 (2012) Xiv:1208.3710v3 [hep-ex]

This new technique is used in CDF and CMS (for muons and electrons). It is currently used in CMS to get a precise measurement of the Higgs mass.

Step 1 : Remove the correlations between the scale for the two leptons by getting an initial calibration using Z events and requiring that the mean $\langle 1/P_T \rangle$ of each lepton in bins of η , Φ and charge be correct.

Step2: The Z mass is used as a calibration. The method requires that the Z mass as a function of η , Φ , or charge of each lepton be correct. Extract fine tuned corrections in bins of η , Φ and charge

After corrections, the Z mass as a function of η , Φ , charge for both the data and hit level MC agree with the generator level Monte Carlo (smeared by resolution, and with experimental acceptance cuts). All charge bias is removed

	Stat.	Error in $\sin^2\theta_{\text{eff}}$	Error in $\sin^2\theta_{\text{eff}}$ from momentum/energy scale:
CMS (2011)		+/-0.00200	+/-0.00130 (prior to using EJC-2012)
ATLAS (2013)		+/-0.00040	+/-0.00050
CDF (2014)		+/-0.00090	+/-0.00005 (using EPJC-2012 method)



3rd innovation: **Event weighting method for A_{FB} analyses**

A. Bodek, Euro. Phys. J. C67, 321 (2010). arXiv:0911.2850v4 [hep-ex]

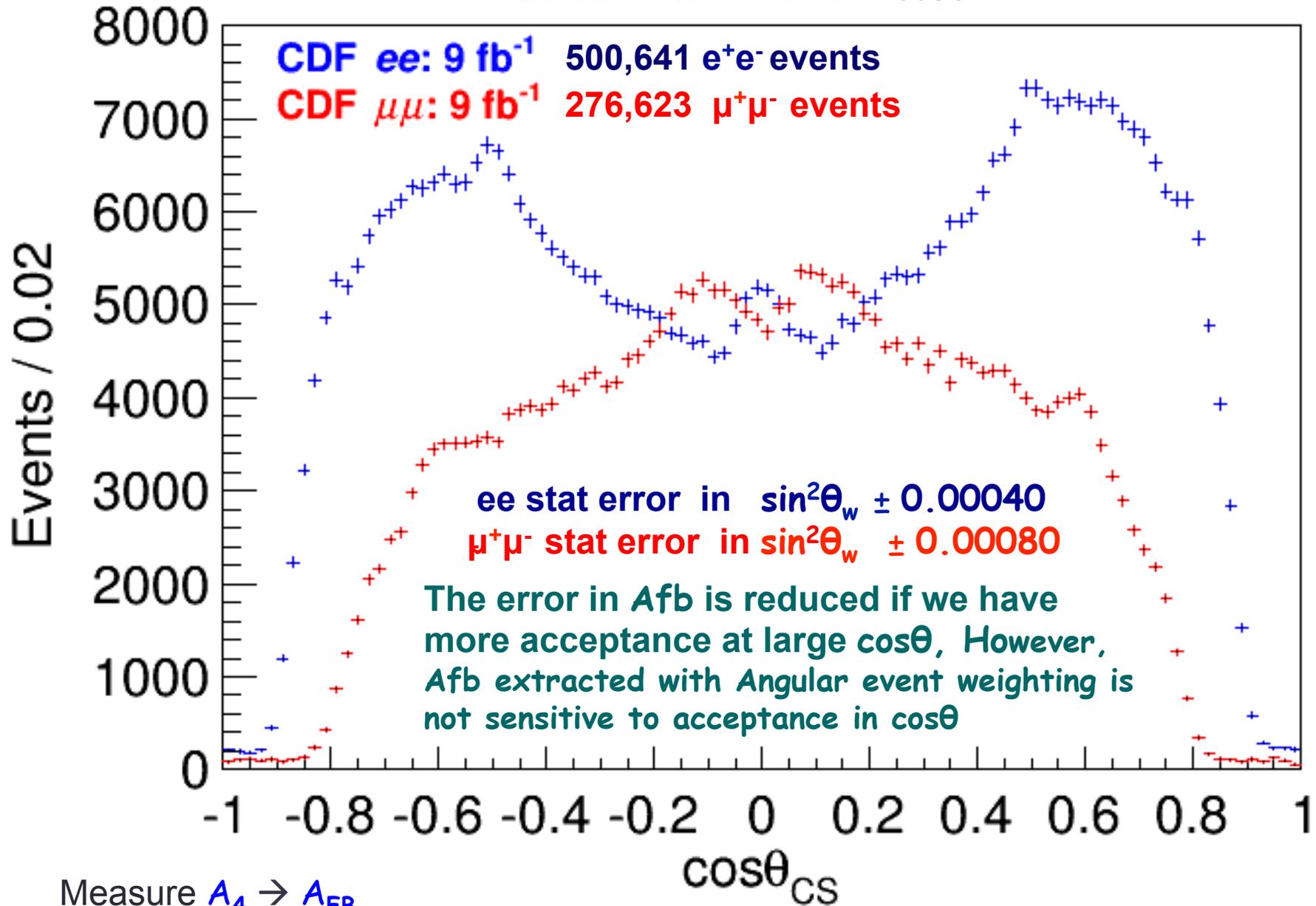
$$dN/d\cos\theta = 1 + \cos^2\theta + A_0(M, P_T) (1 - 3\cos^2\theta)/2 + A_4(M) \cos\theta$$

Angular event weighting is equivalent to extraction of $A_4(M)$ in bins of $\cos\theta$, and averaging the results. It is done all at once using event weights. Events at large $\cos\theta$ provide better determination of A_4 , so they are weighted more than events at small $\cos\theta$. (events $\cos\theta=0$ have zero weight).

In this technique, all $\cos\theta$ acceptance and efficiencies cancel to first order and the statistical errors are 20% smaller. $A_{fb} = (3/8)A_4$

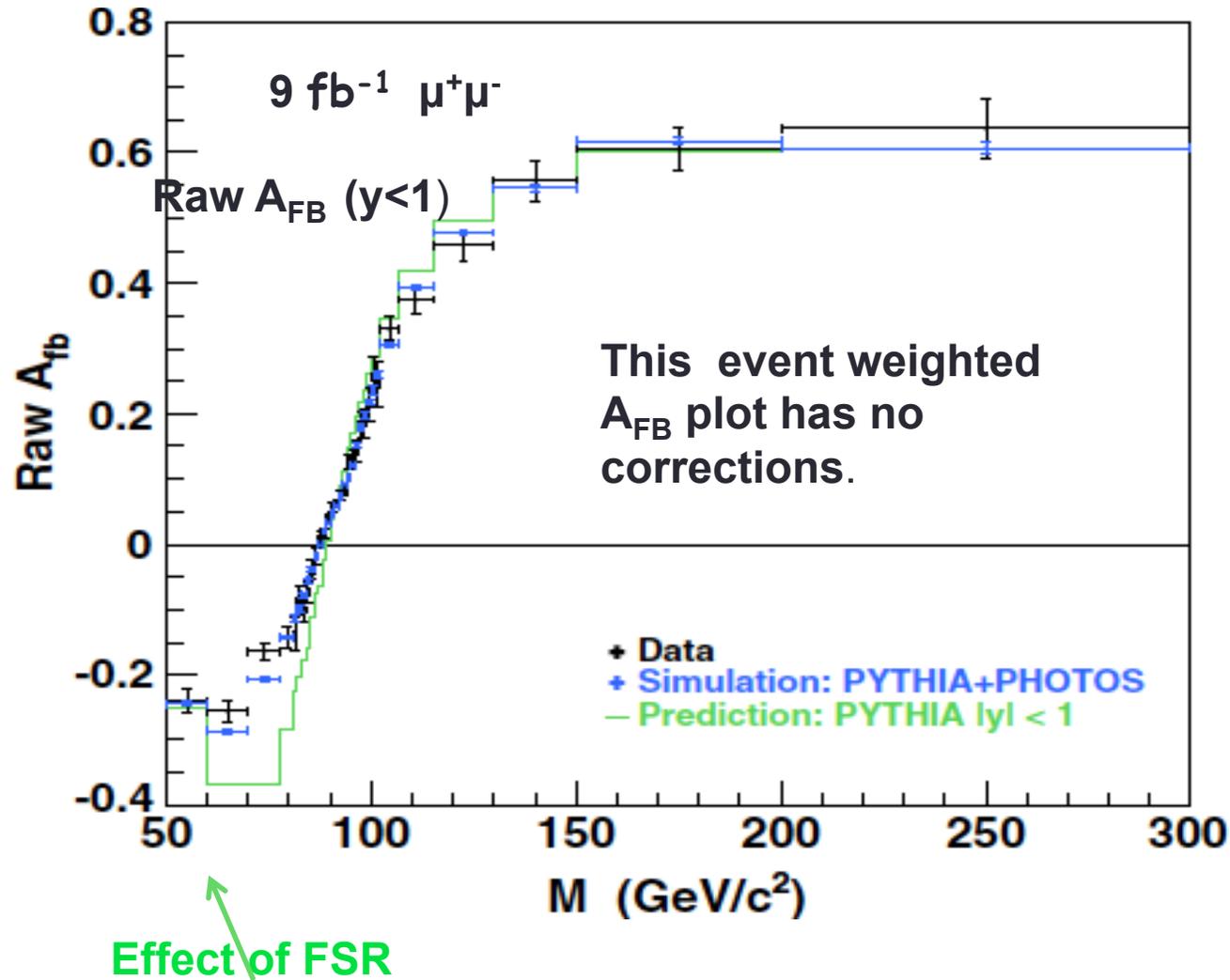
There are three kinds of event weighting can be used, (1) **angular weighting** (2) **dilution weighting**, (3) **both**. In the CDF analysis, angular event weighting is used (since dilution from antiquarks is small) (For LHC need to do both).

Distribution of events in $\cos\theta$



Measure $A_4 \rightarrow A_{FB}$

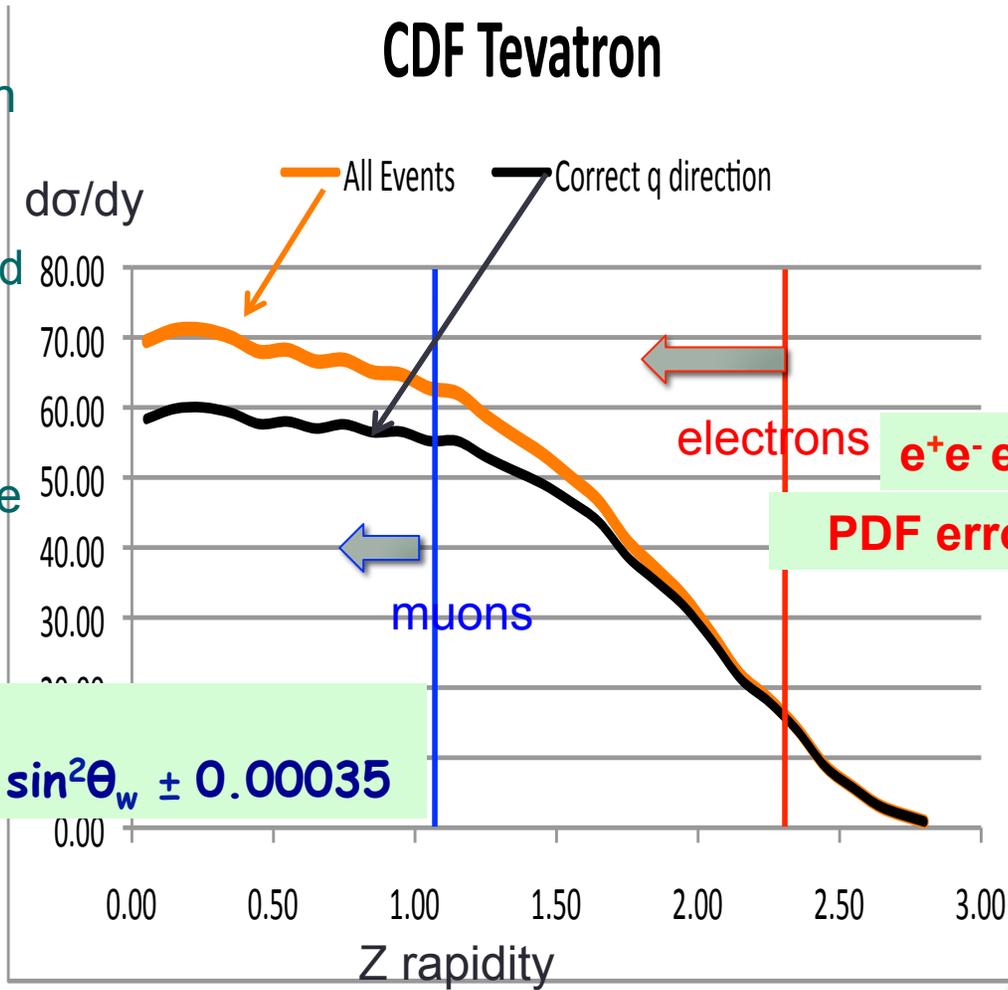
Raw event weighted Afb (No corrections)



Antiquark Dilution

The measured A_{fb} depend on the coverage in rapidity, Sometimes the quark direction is not in the direction of the proton. This small dilution effect depends on the antiquark distributions i.e. on PDFs. (we used CT10), and the rapidity range of the data.

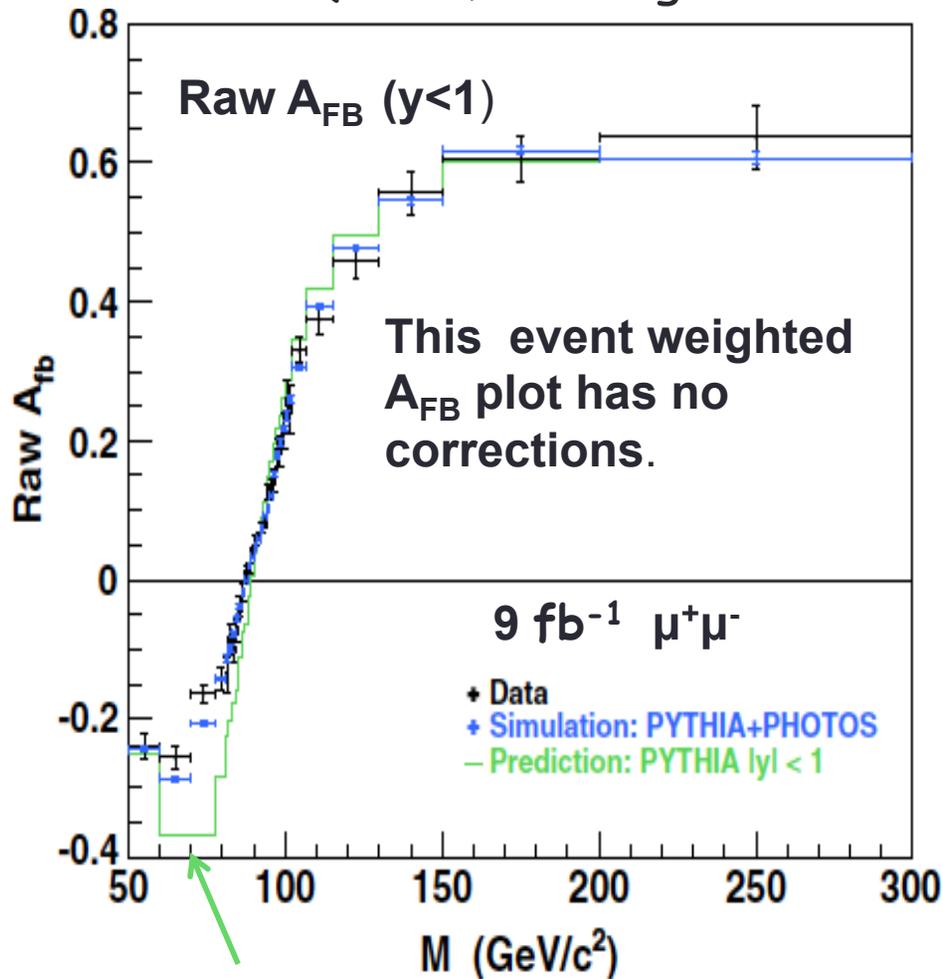
If an additional dilution correction is included in the event weights than the extracted A_{fb} is also independent of the acceptance in rapidity. (more important for the LHC)



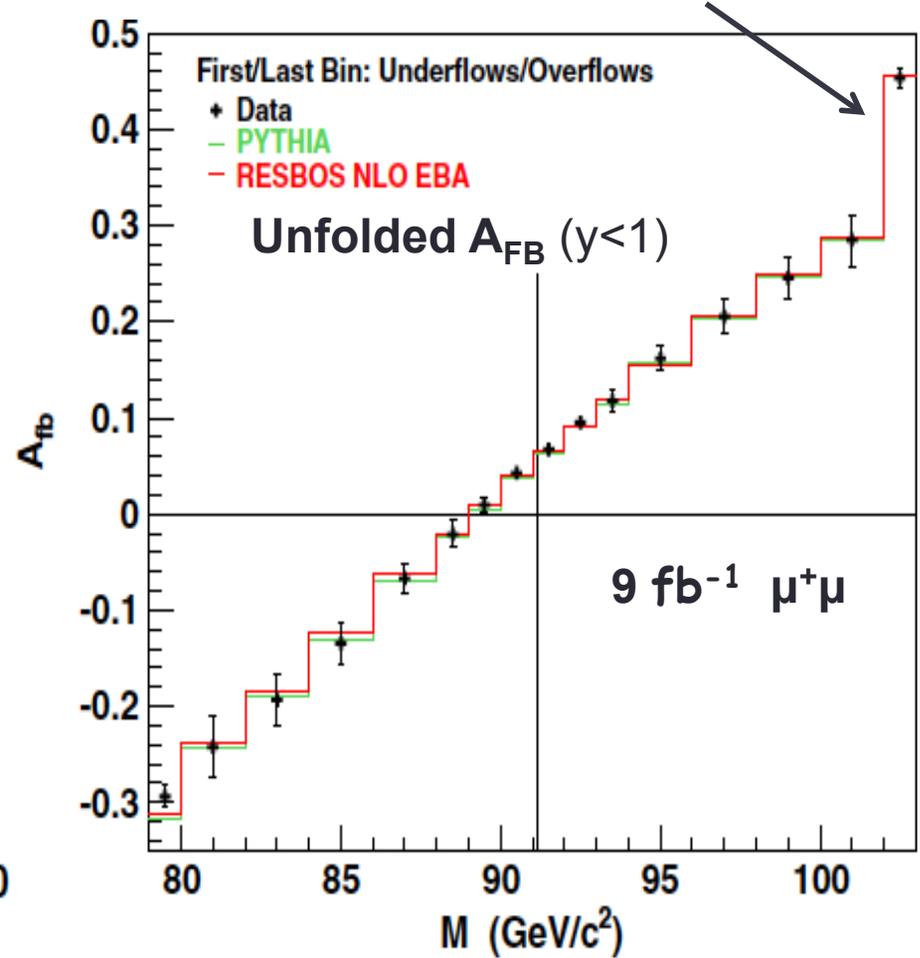
PDF dilution error can be further reduced with better PDFs as LHC data is included in newer PDF sets.

QED FSR and detector resolution smear events between mass bins. We correct for this smearing using matrix unfolding.

(Here, the edge bins are underflow and overflow bins)



Effect of FSR



INPUT M_W to RESBO- EBA or POWHEG-EBA

MH = 125 GeV

$$\sin^2\theta_w = 1 - M_W^2 / M_Z^2$$

EW Rad Corrections yield

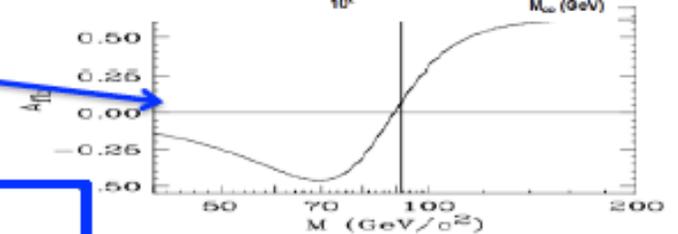
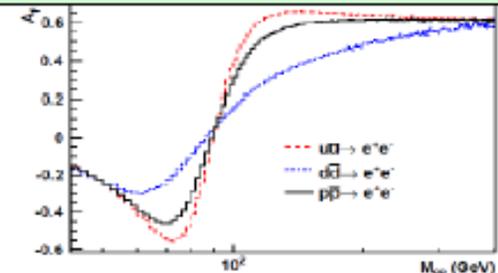
$$\sin^2\theta_{\text{eff}}^{\text{leponic}}(s)$$

$$\sin^2\theta_{\text{eff}}^{\text{u-type}}(s), \sin^2\theta_{\text{eff}}^{\text{d-type}}(s)$$

PDFs + QCD predict $A_4(s)$

$$\text{Predicted } A_{\text{FB}}(s) = (3/8) A_4(s)$$

CDF SM analysis with Full ZFITTER EBA rad corrections analogous to LEP analysis



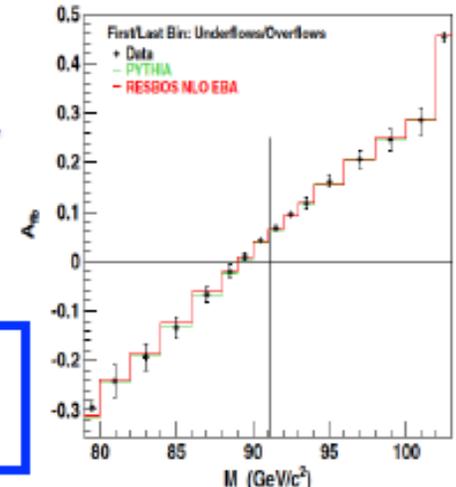
Compare Predicted $A_{\text{FB}}(s)$ to $A_{\text{FB}}(s)$ experiment (unfolded for resolution and FSR) to extract the $\sin^2\theta_w$ that describes the data best

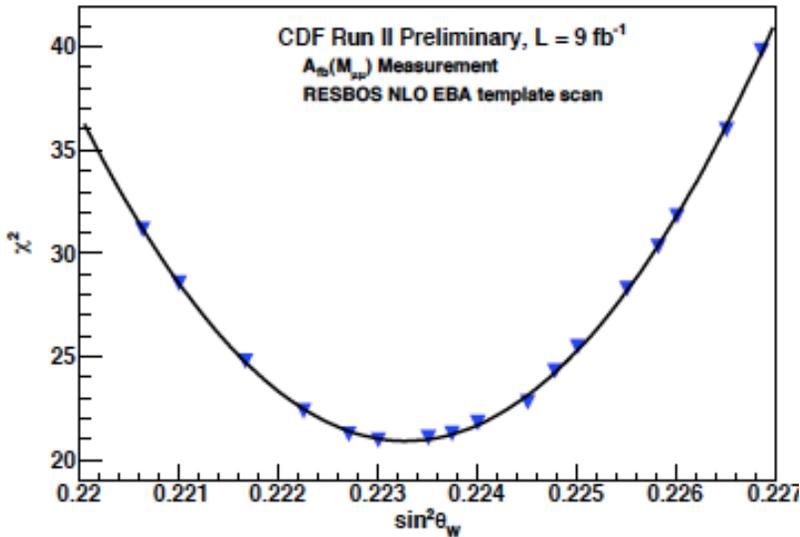
$\sin^2\theta_w = 1 - M_W^2 / M_Z^2$ is used to get a measured M_W indirect

In addition

$$\sin^2\theta_{\text{eff}}^{\text{lept}}(M_Z) \equiv \text{Re } \kappa(M_Z, \sin^2\theta_w) \sin^2\theta_w = 1.037 \sin^2\theta_w$$

Is used to compare to $\sin^2\theta_{\text{eff}}^{\text{lept}}(M_Z)$ at LEP





Source	$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$\sin^2 \theta_W$
Momentum scale	± 0.00005	± 0.00005
Backgrounds	± 0.00010	± 0.00010
QCD scales	± 0.00003	± 0.00003
CT10 PDFs	± 0.00037	± 0.00036
EBA	± 0.00012	± 0.00012

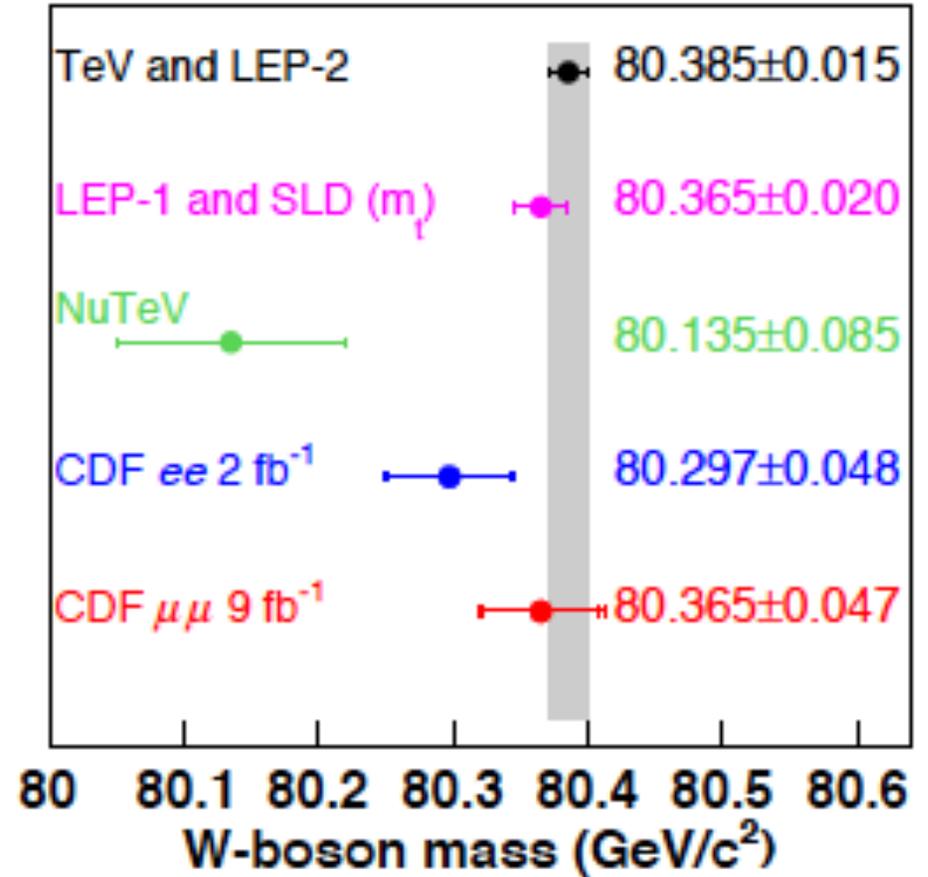
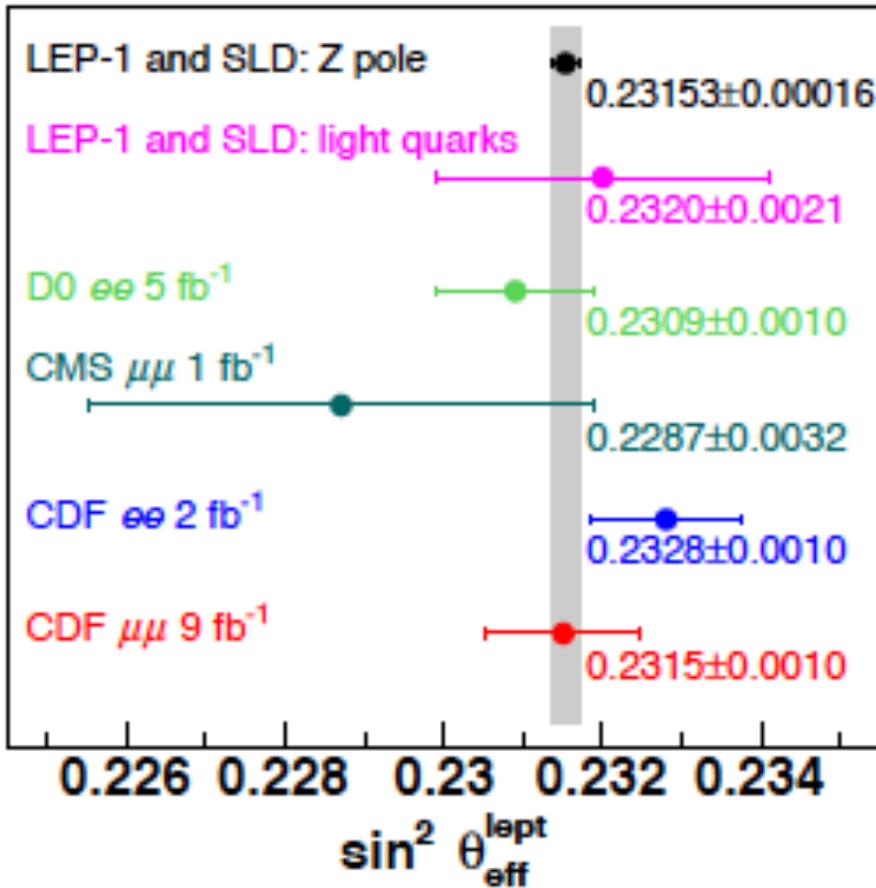
$$\text{ResBos } \sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23150 \pm 0.00090 \pm 0.00011 \pm 0.00035 \text{ (CT10 PDFs)}$$

$$\sin^2 \theta_W = 0.22330 \pm 0.00080 \pm 0.00011 \pm 0.00035 \text{ CT10(PDFs)}$$

$$M_W = 80.365 \pm 0.043 \pm 0.005 \pm 0.018 \text{ (CT10 PDFs)}$$

Template (measurement)	$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$\sin^2 \theta_W$	$\bar{\chi}^2$
RESBOS NLO Full ZFITTER EBA	0.2315 ± 0.0009	0.2233 ± 0.0008	21.1
POWHEG-BOX NLO Full ZFITTER EBA	0.2314 ± 0.0009	0.2231 ± 0.0008	21.4
Tree LO Full ZFITTER EBA	0.2316 ± 0.0008	0.2234 ± 0.0008	24.2
PYTHIA No EW radiative cor. CT5L	0.2311 ± 0.0008	...	20.8

Comparison to other measurements



* A factor of 2 reduction in errors is expected in Fall 2014 when the analysis of the CDF $e+e-$ (9 fb^{-1}) data is completed.

Conclusion and more results in the near Future

- CDF ($\mu^+\mu^-$ 9 fb⁻¹) published 2014 EBA rad corr CT10 PDFs
- $M_w(\text{indirect}) = 80.365 \pm 0.045 \text{ GeV (2014)}$

$$\sin^2\theta_{\text{eff}} = 0.23150 \pm 0.00090(\text{stat}) \pm 0.00011(\text{sys}) \pm 0.00035 \text{ (PDF)}$$

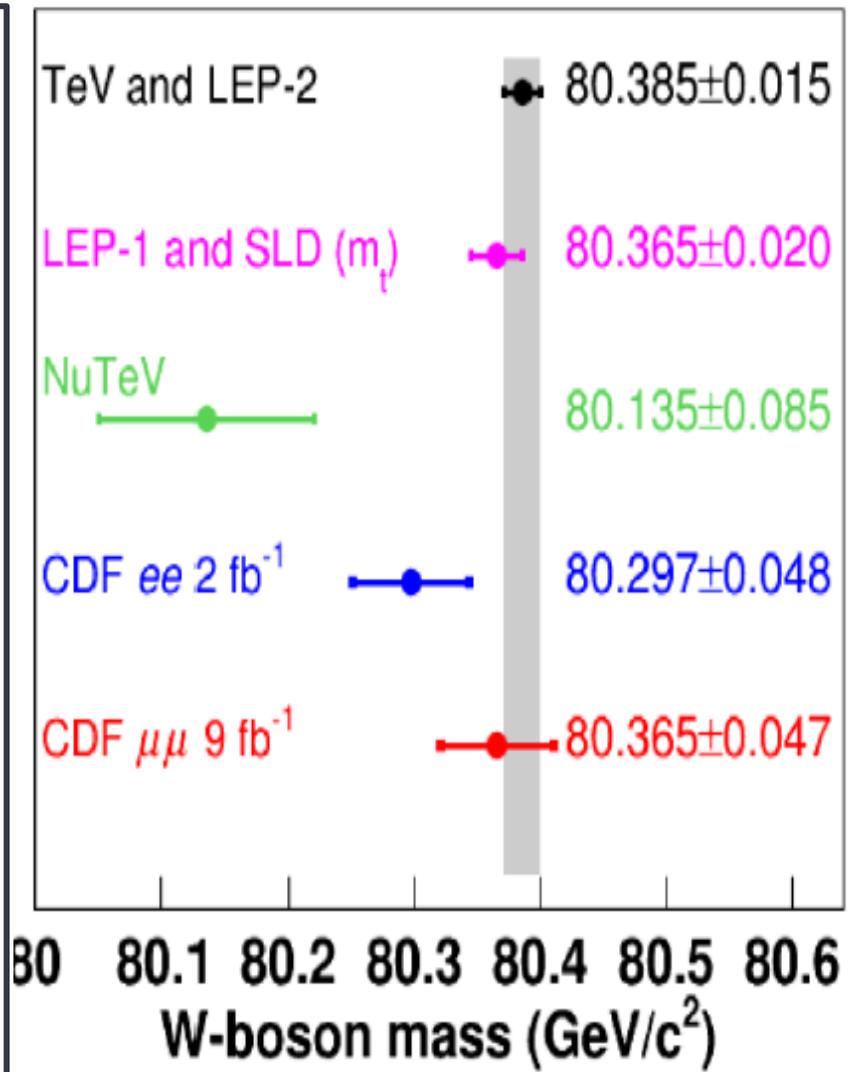
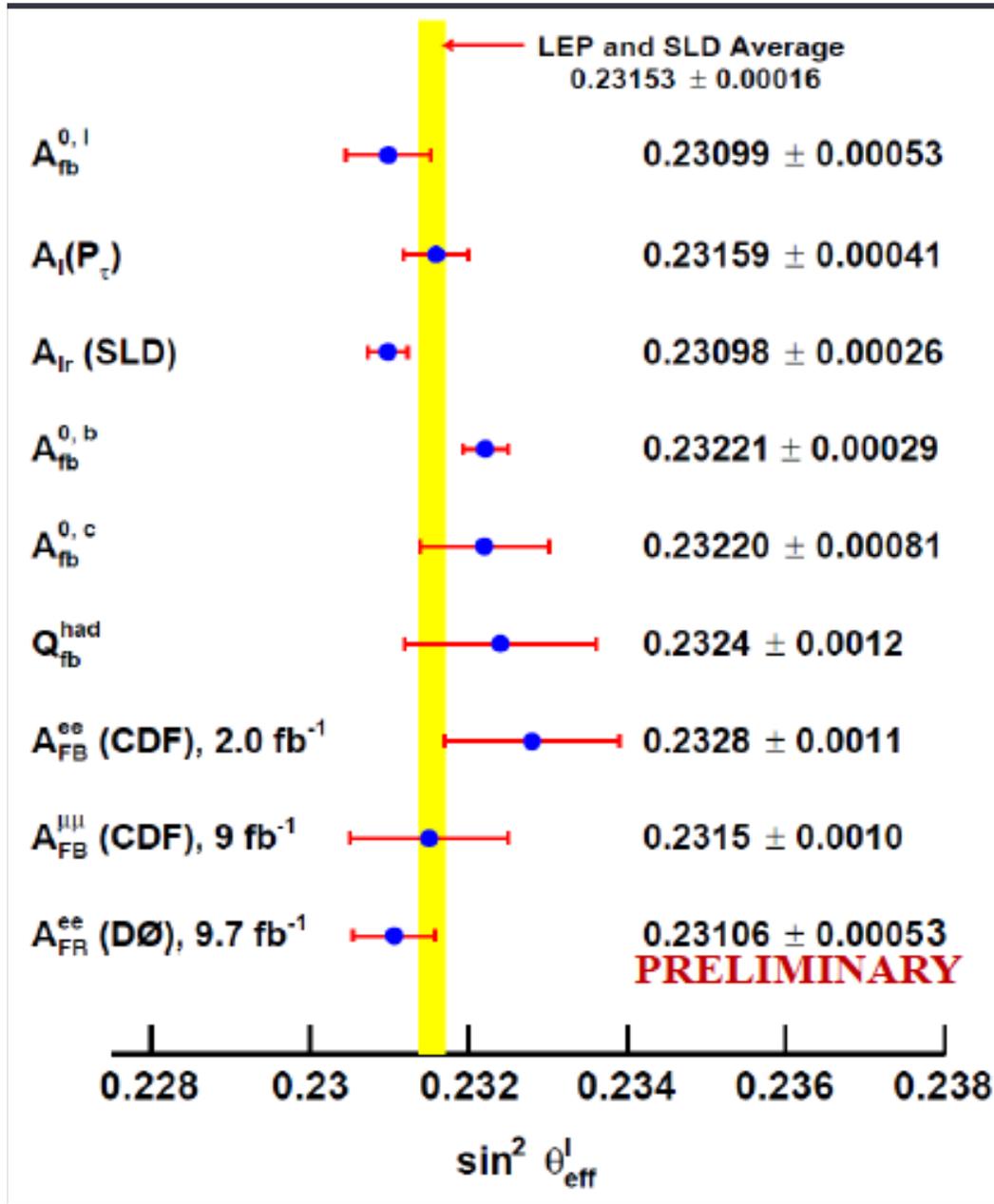
Expected results in Fall 2014 CDF (e^+e^-) 10 fb⁻¹ (errors reduced by 1/2)

- $\sin^2\theta_{\text{eff}} = 0.23xxx \pm 0.00040(\text{stat})$
- $\pm 0.00005(\text{sys})$
- $\pm 0.00029 \text{ (PDF-CT10)}$
- .
- We also expect PDFs to improve 2014
- Versus

- LEP 0.23098 ± 0.00026
- SLD 0.23221 ± 0.00029

- End of 2014 CDF and D0 combined will match LEP/SLD errors.
Indirect and direct measurements of M_w will have comparable errors.

Additional Slides



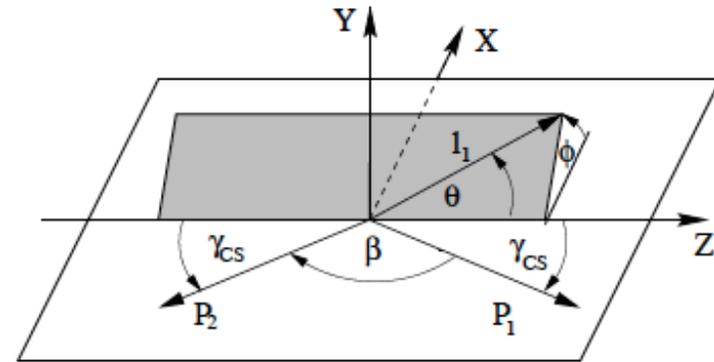
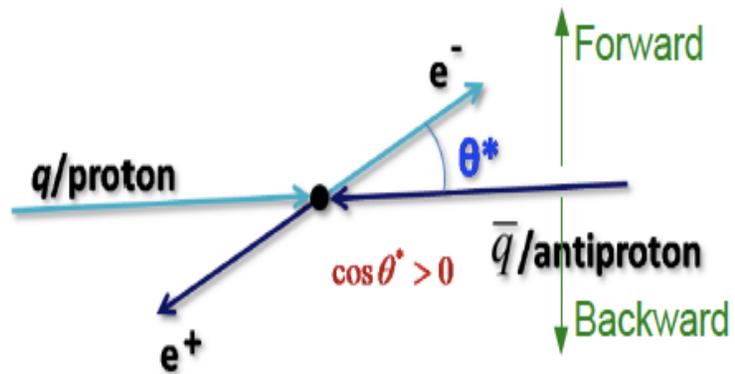
Conclusion: $\sin^2\theta_w$ and indirect measurements of M_w .

A: CDF Phys. Rev D. 89, 072005 (2014) Run II 9 fb⁻¹ $\mu^+\mu^-$
 Reports three measurements with statistical errors of
 $\sin^2\theta_{\text{eff}}$ (± 0.00090) $\sin^2\theta_w$ (± 0.00080) M_w^{indirect} (± 44 MeV)

B: CDF: Expected Fall 2014 full Run II data set 9.7 fb⁻¹ e^+e^-
 will have three measurements with statistical errors of
 $\sin^2\theta_{\text{eff}}$ (± 0.00044), $\sin^2\theta_w$ (± 0.00040) and M_w (indirect) (± 22 MeV)

Which means that a measurement of M_w (indirect) with the combined CDF/D0 9 fb⁻¹ run II data would have a statistical error of ± 15 MeV, which is equal to the ± 15 MeV error in average of all world measurements of M_w (direct)

In addition, it would address the LEP-SLD Difference. LEP SLD difference is 0.00122



Drell-Yan asymmetry is measured in the Collins-Soper frame. The Collins-Soper frame is the CM frame of the dilepton pair. It is also the q-qbar center of mass

The dilepton pair can have P_T in the laboratory frame. The P_T could originate from gluon emission by the quark in the proton, or by the antiquark in the antiproton (and also from a qG process). Therefore, unlike e^+e^- collisions at LEP, the q and $qbar$ are not collinear in the lab.

The $P_T=0$ Born level polar angle distribution: $1 + \cos^2\theta + A_4 \cos\theta$

Is replaced with $1 + \cos^2\theta + A_0(M, P_T) (1 - 3\cos^2\theta)/2 + A_4 \cos\theta$

For dileptons with a P_T , the small change in the $\cos\theta$ distribution in the Collins-Soper frame is well understood. CDF has measured this change and it agrees with POWHEG QCD prediction. It is accounted for in the analysis and does not have much impact to the results.

$$\frac{dN}{d\Omega} \propto (1 + \cos^2\theta) + A_4 \cos\theta + A_0(M, P_T) (1 - 3\cos^2\theta)/2$$

$$A_0 = A_2 = \frac{kP_T^2}{kP_T^2 + M^2}$$

K=1.65 at the Tevatron
(higher at the LHC since more qG)

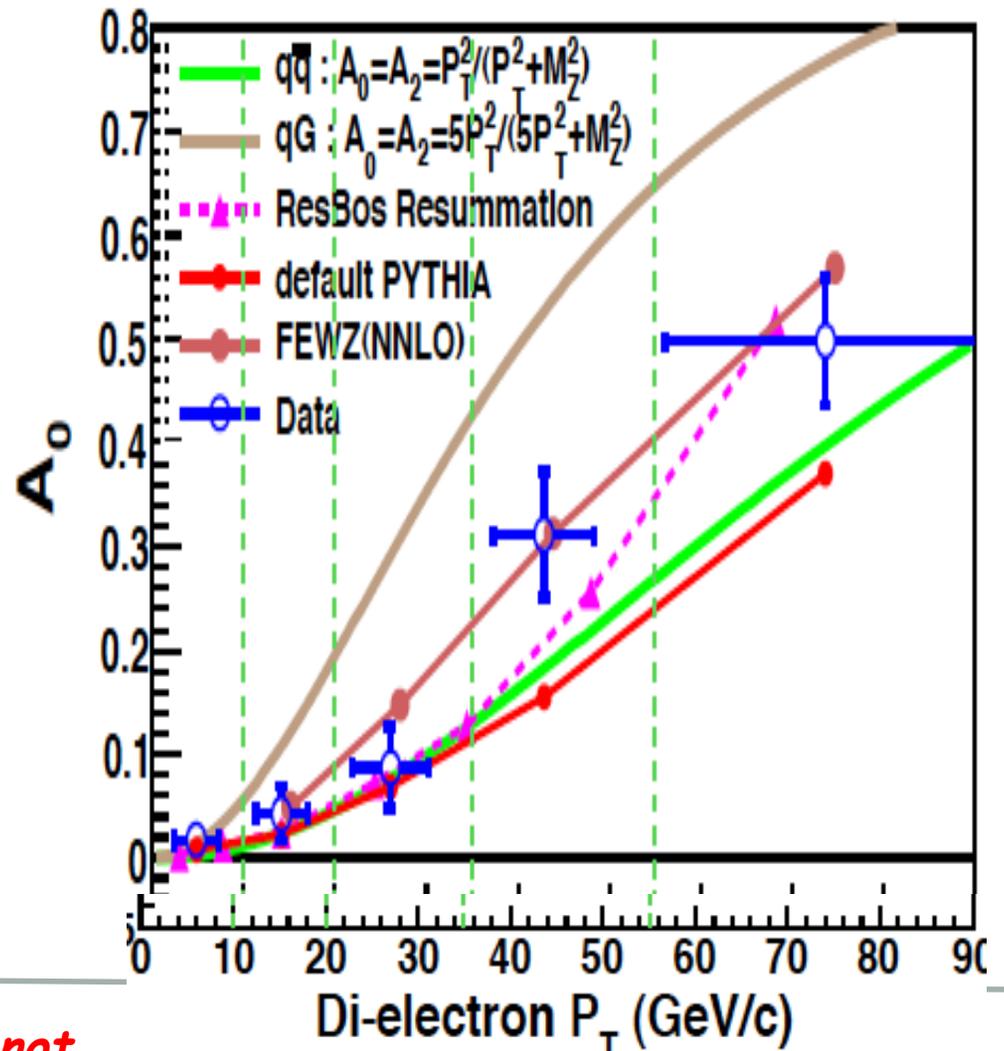
The NLO QCD correction to the angular distribution are taken into account in POWHEG and RESBOS.

The correction is fully taken into account in the CDF Afb analysis.

However it does not make a significant difference.

This is good because PYTHIA does not have the correct angular distribution (only q-qbar no qG)

CDF Phys Rev, Lett 106 ,24180 (2011)



- In the CDF analysis, we calculate $A_4(M)$ for various values of the $\sin^2\theta_W$ model parameter and compare it to the measurement
 - ZFITTER EBA techniques and complex valued **form factors** (ρ , κ) are used
 - Quark-loop corrections to the photon propagator ($1-\Delta\alpha(s)$) are used
 - Real part is the running EM coupling : $\text{Re } \Delta\alpha(Mz) \approx 0.06$ (1/128)
 - Imaginary part is non-zero and is used: $\text{Im } \Delta\alpha(Mz) \approx -0.02$
 - Complex valued corrections are incorporated into the Drell-Yan amplitude
- A_4 is directly related to a mix of $\sin^2\theta_{\text{eff}}$ from the lepton, d-, and u-type quarks
 - The best fit value, $\sin^2\theta_W$, is indirectly related to A_4 and model dependent
 - Model is almost identical to the one derived from Z-pole fits at LEP
 - We use $m_H = 125 \text{ GeV}$ (LHC value, but consistent with LEP fit value)
 - $\sin^2\theta_{\text{eff}}(M) = \kappa(M) \sin^2\theta_W$: this product is model independent
 - We provide the leptonic $\sin^2\theta_{\text{eff}}^{\text{lept}}$ at the Z-pole for comparison with the LEP

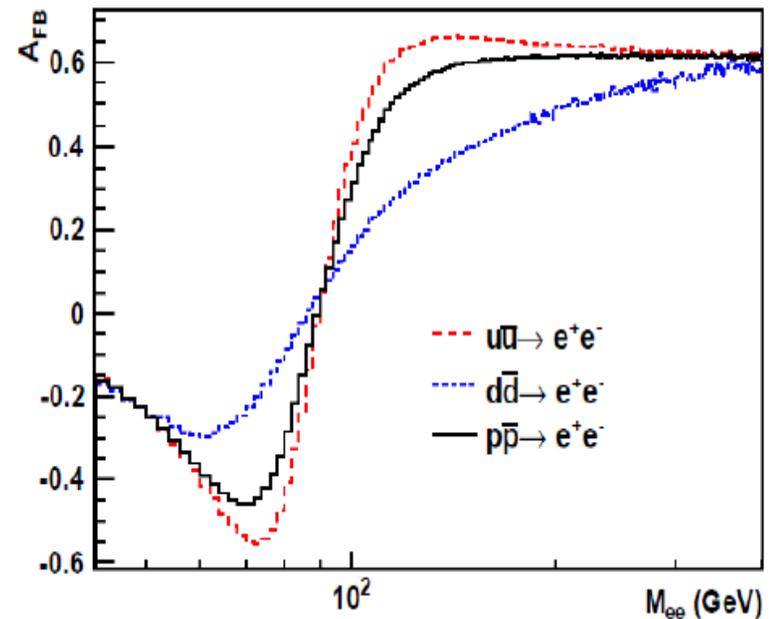
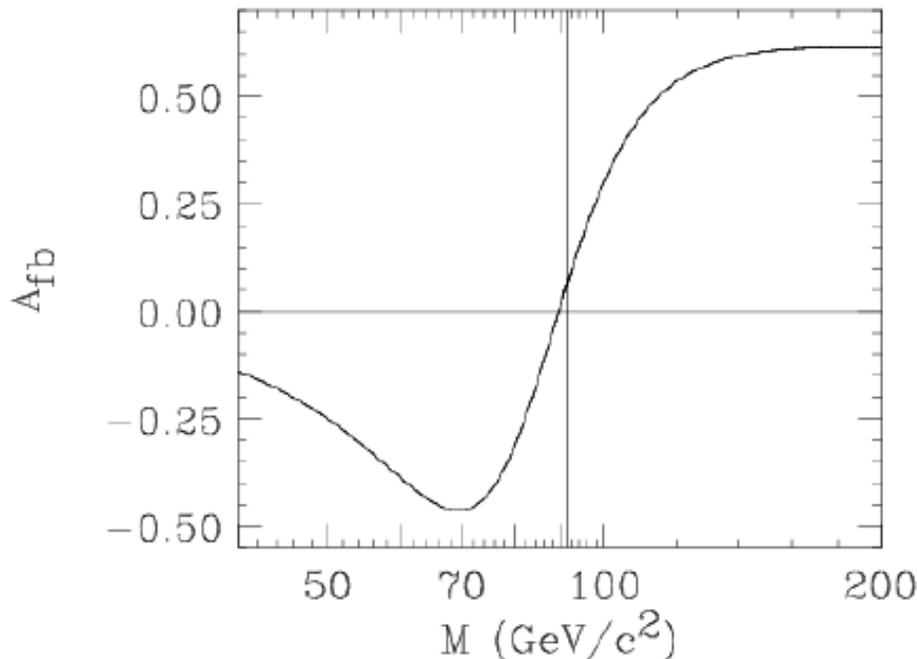
$$\sin^2\theta_{\text{eff}}^{\text{lept}} \equiv \text{Re } \kappa(Mz, \sin^2\theta_W) \sin^2\theta_W = 1.037 \sin^2\theta_W$$



$$A_{f_{FB}}(M) = (3/8)A_4(M)$$

- Vertical line is $M = M_Z$ where γ/Z interference is zero
- γ/Z interference $\propto (s - M_Z^2)$
 gets large away from Z peak and dominates
 related to g_A and no direct dependence on $\sin^2\theta_W$

More details in Phys. Rev. D88, 072002 (2013)



The full EW ZFITTER modification (Enhanced Born Approx - EBA) were incorporated into two QCD calculation of A_{FB} (e.g. POWHEG, RESBOS) with CT10 NLO PDFs, and also in a stand alone (LO) calculation.

The calculations have only one parameter, i.e on shell $\sin^2\theta_w$.
 We find the $\sin^2\theta_w$ value which the model fits the data for $A_4(M)$
 We then have $\sin^2\theta_{eff}^{lept}(M_Z) = 1.037 \sin^2\theta_w$

Without ZFITTER EBA corrections, the input to POWHEG, RESBOS is just $\sin^2\theta_{eff}^{lept}$ which is assumed to be independent of M . In this case, since no EW radiative corrections are applied, the $\sin^2\theta_{eff}^{lept}$ which fits the data is an average which depends on the range of M that is being used.

With the Full ZFITTER EBA radiative corrections the extracted value of $\sin^2\theta_{eff}^{lept}$ is larger by the following amounts.

RESBOS NLO +EBA template -	- by 0.00031
POWHEG-BOX NLO +EBA template	- by 0.00021
LO template +EBA	- by 0.00047

Full ZFITTER EBA EW radiative corrections

TABLE III. Extracted values of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ and $\sin^2 \theta_W$ for the EBA-based QCD templates. The PYTHIA entry is the value from the scan over non-EBA templates calculated by PYTHIA 6.4 with CTEQ5L PDFs. The uncertainties of the template scans are the measurement uncertainties ($\bar{\sigma}$). Other measurements are listed in parentheses.

Template (measurement)	$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$\sin^2 \theta_W$	$\bar{\chi}^2$
RESBOS NLO Full ZFITTER EBA →	0.2315 ± 0.0009	0.2233 ± 0.0008	21.1
POWHEG-BOX NLO Full ZFITTER EBA	0.2314 ± 0.0009	0.2231 ± 0.0008	21.4
Tree LO Full ZFITTER EBA	0.2316 ± 0.0008	0.2234 ± 0.0008	24.2
PYTHIA NO EBA →	0.2311 ± 0.0008	...	20.8

RESBOS NLO templates with full ZFITTER EBA EW rad correction yield a value of $\sin^2 \theta_{\text{eff}}$ which is **0.00040** larger than the values extracted using PYTHIA templates with CTEQ5L PDFs with no radiative corrections.

Investigating EW radiative corrections,

POWHEG+full ZFITTER EBA rad cor (private CDF version)

RESBOS+full ZFITTER EBA rad cor (private CDF version)

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POWHEG (which is a MC) has a new version with EW radiative corrections - Eur.Phys.J. C73 (2013) 2474, arXiv:1302.4606.

(we are currently testing this version)

HORRACE and Zgrad (not full EBA)

FEWZ3.1 also has EW radiative corrections (it is not a MC)

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